Type 2 Fuzzy Logic System in Water Network Optimal Design Procedure

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Abstract. In the past decades, Type 2 Fuzzy Logic System (T2 FLS) has been widely used in engineering applications. To outperform the Type 1 FLSs, T2 can be applied with dynamic uncertainties. In the present work, the Genetic Algorithm (GA) is used to design the least cost Water Distribution Networks (WDN). The performance of each GA generated network is done using iterative T2 FLS. This novel method enables the user to insert the imprecision in hydraulic criteria for the optimal design procedure. The GA based design algorithm of the two loop network from past literature is discussed in details where the uncertainty in head at network nodes are considered using upper and lower membership functions for each head linguistic variables. The designed network properties indicate the need to use Interval T2 FLS in hydraulic engineering problems.

Keywords: component; fuzzy logic system; interval type two fuzzy; genetic algorithm; water distribution networks

1. Introduction

The Water Distribution Networks (WDNs) design covers the complicated combination of simulation and optimization. Due to the high efficiency of its application in multi-objective problems, Genetic Algorithm (GA) has been used in several Water Distribution Networks Design [1-4]. The GA based optimal design procedure of a WDN is a cost objective function which is restricted by topologic, geometric, and hydraulic constraints. Despite using crisp logic to make decision in GA, considering the matter of uncertainty is vital for an algorithm to have practical results. In recent years, Fuzzy Design System (FDS) has been practiced in few WDN design. In [5] Fuzzy System (FS) is used to implement the imprecision in definition of design loading condition and roughness coefficient of pipes during hydraulic analysis of networks. In [6] FS is used to describe hydraulic constraints in a dynamic programming based optimal WDN design procedure. They introduced a multi objective optimization system in which the first objective function was the network's cost while the second one was FS to calculate the benefit of each designed network. In [7] a Fuzzy Genetic Algorithm (FGA) is introduced which is a GA based search method in which fitness evaluation is carried out using FDS. A fully FDS system consists of input and output membership functions and a rule set is used to simulate the tolerance in constraint deviation in network elements.

In the previous published works, the Type 1 Fuzzy Logic System (T1 FLS) has been employed in which the membership functions (MFs) can either be chosen based on the users' opinion. Therefore, the MFs from two individuals could be quite different based on their experiences, perspectives, etc. Despite the T1 FLS name carries the connotation of uncertainty, there are limitations of T1 FLS to model the uncertainty regarding the crisp value in its membership grades. Recently, the Type 2 Fuzzy Logic System (T2 FLS) increasingly is being used by practical engineers [8-12] where the MFs are themselves fuzzy. In other words, where the T1 FLS is capable of simulating system vagueness, the T2 FLS enhance user to consider system uncertainty.
In this paper, the T2 FLS is used in a GA based optimal WDN design procedure. Using this novel system improves the current WDN design by considering both the vagueness and uncertainty during design and operation periods. The paper begins with brief introduction to FGA and T2 FLS. It is followed by theoretical foundation of T2 FLS implementation in WDN design systems. A detailed study of proposed method's application using the well known two looped network in [13] shows the strength of it in uncertainty modeling.

1.1. FGA
The genetic evolutionary algorithm stems from the natural random selection process, which selects population of individuals as a set of solutions and iterates through generations to reach the near optimal solution. As previously discussed, the GA uses crisp decision system to progress toward the optimal solution.

1.2. Type 2 Fuzzy Logic System (T2 FLS)
The T2 FLS is originally proposed by in [14]. This algorithm demonstrates the ability to perform better in facing dynamic uncertainty, by using fuzzy membership degree. This new uncertainty dimension provides additional freedom to model the dynamic input and output uncertainties. The interval type 2 fuzzy logic system (IT2 FLS) is a special method of T2 FLS which is currently more widely used due to its reduced computational complexity and cost. An example of IT2 FLS MF is presented in Fig. 1, which is bounded by \( X_u \) and \( X_l \) as upper MF (UMF) and lower MF (LMF), respectively. The area between \( X_u \) and \( X_l \) is called the footprint of uncertainty (FOU). The construction of MFs is executed by surveys, optimisation techniques, judgment, experience and opinion, training data, etc. Fig. 2 shows the schematic diagram of IT2 FLS.

Each fuzzy linguistic variable as \( X \) is defined by its membership value as \( \mu_X(x, \mu) \), i.e.

\[
X = \int_{x \in D_x} \int_{u \in J_x \subseteq [0, 1]} \frac{\mu_X (x, u)}{(1/u)/x}
\]

(1)

where \( x \) is the primary variable in \( D_x \) domain, \( u \in [0,1] \) is the secondary variable in domain \( J_x \subseteq [0,1] \) at each \( u \in D_x \). Uncertainty about \( X \) is covered by the union of primary variables as FOU for \( x \), i.e.

\[
\text{FOU}(X) = \cup_{x \in D_x} J_x \{(x, u): u \in J_x \subseteq [0,1]\}
\]

(2)

The size of FOU is directly related to the uncertainty and it can be defined by only two UMF and LMF as

\[
\text{FOU}(X) = \cup_{x \in D_x} \left( \mu_{U,X}(x), \mu_{L,X}(x) \right)
\]

(3)
where $\mu_{U}(x)$ and $\mu_{L}(x)$ are upper and lower membership values. To obtain the crisp output value, the IT2 fuzzy output set $B$, first must be type reduced and then difuzzified. The centroid of $B$ is defined as

$$C_B = \int_{\theta_1}^{\theta_2} ... \int_{\theta_1}^{\theta_2} \frac{1}{\theta_1 \theta_2 \sum_{i=1}^{N} \theta_i} = [y_L, y_R]$$

where $N$ is the number of rules and $\theta_i$ is the $B$ membership value at $x_i$. The centroid $C_B$ is an interval fuzzy set which is described by its left and right end points $y_L, y_R$ [15]. In $y_L, y_R$ computing procedure, the two $L$ and $R$ switching points should be calculated by Karnik-Mendel (KM) iterative procedure. The final crisp difuzzified value $y$ is computed as [12]

$$y = (y_L + y_R)/2$$

3. Model Development

The WDN optimization aims to design the least-cost network under different range of constraints. Here, the GA objective function is defined as

$$C(D, L) = \sum_{k=1}^{NP} c_k \cdot (d_k) L_k$$

where $D$ and $L$ are vectors defining diameter and length of pipes, respectively, $c_k(d_k)$ is the cost of pipe with diameter $d_k$ per unit length, and $NP$ is the number of pipes. Generally in GA designed networks, head deviation constraint in network nodes (consumers) is the most important hydraulic constraint defined as

$$\text{diffHead} = \sum_{i=1}^{NJ} [H_{min} - H(i)] \quad \text{if} \quad H(i) \leq H_{min}$$

where $\text{diffHead}$ and $H_{min}$ are the head deviation constraint and minimum required head by consumers, respectively, $H(i)$ is the head in node $i$, and $NJ$ is the number of network nodes. To use FGA optimization technique, the constraint deviation is added to objective function as a penalty function. Therefore, Eq. 7 defines as

$$C(D, L) = \sum_{k=1}^{NP} c_k \cdot (d_k) L_k + \sum_{k=1}^{NC} \text{PEN}_k \times \text{CON}_k$$

where $\text{PEN}_k$ and $\text{CON}_k$ are the fuzzy representative of constraint $k$ and a constant number related to constraint $k$, respectively, and $NC$ is the number of constraints.

4. Model Application

The two loop network introduced in [13] has been designed by many optimization techniques. The network consists of 8 pipes arranged in two loops, is fed by 210 m head storage. Fig. 3 shows the predefined network layout in addition to nodes' altitude and the consumers' demand.

The pipes are all 1000 m long and through the optimization process, each pipe may be divided into two segments where each segment length and diameter should be defined. The 25.4, 50.8, 76.2, 101.6, 152.4, 203.2, 254, 304.8, 355.6, 406.4, 457.2, 508, 558.8, and 609.6 mm diameter available pipes cost 2, 5, 8, 11, 16, 23, 32, 50, 60, 90, 130, 170, 300, and 350 with arbitrary units per unit length. The Hazen-Williams coefficient for all pipes is assumed to be 130. The required head as reported in literature is 30 m.

For the imprecision in the system status such as the pipes’ roughness coefficient and consumers demand which could be called as system vagueness, T1 FLS is employed to decide on networks’ quality through generations based on fuzzy logic. The uncertainty in required head by consumers could not be modelled by T1 FLS. Regarding the WDN design standards, 30 m head should be delivered to consumers in urban areas. In this viewpoint, all consumers are treated equally without any consideration of tall buildings or one-story houses. The uncertainly in required head could be modelled by IT2 FLS using fuzzy parameter $P$ as

$$P = 30/min\{H\}$$

where $\min(H)$ is the minimum head observed in network nodes. Three head linguistic variables as bad, medium and good are introduced. The UMF, LMF and FOU for each linguistic variable are shown in Fig. 4.

The rule set consists of three rules as
• If $x$ is BAD then $y$ is $5(\mu_x^{BAD})^2 + 20$
• If $x$ is MEDIUM then $y$ is $5(\mu_x^{MEDIUM})^2 + 10$
• If $x$ is GOOD then $y$ is $5(\mu_x^{GOOD})^2$

The IT2 FLS using above MFs and rule set forms the decision system of GA based optimal design algorithm.

The external genetic search cycle properties are as follow
• The size of GA population is 200.
• The selection and crossover operators are tournament and one-point methods, respectively.
• Mutation and crossover rates are 0.05 and 0.6, respectively.
• The stop criterion is the stall number of 200 generations (no change in the best fitness function in 200 consecutive generations).
• The elitism is observed by transferring 30 percent of best fitted individuals without any change to next generation.

The least-cost designed network using the proposed algorithm costs 446436 arbitrary units which is approximately 10 percents more expensive than the one designed in [16] (402352 units). The additional cost of designed network by the proposed method, compared to the least cost network in literature, is the consequence of considering uncertainty in desired head in nodes. [16] constrained the algorithm by delivering 30 m head to consumers, while in this paper up to 60 m head is introduced to fulfill consumers preferred head.

The designed network cost is 2 percent more than the result obtained in [7] using T1 FLS. Tables 1 and 2 show the observed heads and designed network properties. The values of heads show the capability of proposed algorithm in introducing a range of heads as the hydraulic criteria. In the previous algorithm, only one value as the desired head can be introduced.

5. Conclusions

In this paper, the IT2 FLS technique has been investigated as the decision system in a built-in GA based optimal WDN design procedure. Considering the uncertainty in desired hydraulic characteristics of network elements boosts the need to use IT2 FLS. Introducing UMF and LMF for each head linguistic variable as the hydraulic constraint bounds in network nodes, defines the FOU of desired head in nodes. Results show that uncertainty could be modelled well using IT2 FLS. The designed network cost is marginally more than previous least cost networks, considering the possibility of more demanded head by some consumers.

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6. References


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Figure 4. Membership functions (MFs) for head linguistic variables.

### TABLE II. PIPE LENGTH AND DIAMETER OF DESIGNED NETWORK

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