Adaptive Sliding Mode Control of a Wastewater Treatment Process inside a Biological Sequencing Batch Reactor

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Abstract—This work deals with the design of a dynamical adaptive Sliding Mode Control (SMC) strategy for a wastewater treatment bioprocess taking place inside a Sequencing Batch Reactor (SBR). The dynamical control law is computed by means of a combination between the exact linearization approach and SMC. To deal with the parametric uncertainties of the nonlinear bioprocess, the adaptive form of SMC law is designed by means of direct, overparameterized adaptive control technique. The control goal is to keep a low level of pollutant concentration. Numerical simulations are included to test the behavior and the performance of the proposed adaptive SMC strategy.

Keywords—bioengineering; control systems; adaptive control; sliding mode control

1. Introduction

The biotechnological processes are strongly nonlinear, and the process parameters are highly uncertain. Also, bioprocesses are characterized by the absence, in most cases, of cheap and reliable instrumentation. In order to cope with these problems, different control strategies were developed, such as adaptive approach (Bastin and Dochain [1]), [2], sliding mode control [3], fuzzy and neural strategies, etc.

The aerobic treatment of wastewater by activated sludge is a common process, but the characteristics of a lot of industrial emissions frequently cause operational problems in continuous flow systems [4], [5]. Thus, discontinuous processes such as biological SBRs can be taken into consideration due to the fact that they could operate better than the continuous processes, in terms of operation costs, process stability, and reliability [2], [4], [5].

SMC has been widely accepted as an efficient method for control of uncertain nonlinear systems [6], [7], [8]. SMC has been shown to be able to achieve very good performance in the presence of parameter uncertainties and external disturbances [6]. Results from the differential algebraic approach to control theory [9], have greatly improved the applicability of discontinuous feedback strategies, especially those of sliding mode type, leading to asymptotic stabilization and tracking in nonlinear systems [6], [7], [8]. Some of traditional disadvantages of SMC are related to the bang-bang character of input signals and the chattering of output and state variables [6], [7]. These difficulties can be circumvented via dynamical SMC while retaining the robustness and simplicity of this class of control strategies. The classical applications of SMC (such as robotics, electrical machines etc.) were extended to chemical processes (see for Llanes-Santiago et al. [8]) and to biochemical processes, with several applications already reported [3].

The adaptive control is a good method for the bioprocesses, but there are some problems when large and abrupt changes occur in process parameters [2]. Among the advantages of SMC are robustness, controller order reduction, disturbance rejection, and insensitivity to parameter variations. The main disadvantage of the SMC strategy is the chattering phenomenon [6], [7], [8].
In this work, a dynamical adaptive sliding mode control is proposed for an aerobic wastewater treatment process inside a SBR. The SMC law is obtained by designing a discontinuous feedback law, by using the exact linearization of the system and by imposing a SMC action that stabilize the output. A problem in the case of SMC is that the robustness of sliding surface with respect to uncertainties is valid only for small parameter variations. If the parameters are imprecisely known, then the stability of the closed loop system can be affected. In that case, an adaptive dynamical SMC law is proposed in order to improve the performance of controlled bioprocess. Another problem can be the unavailability of state measurements. In such cases, some state observers can be used [1].

The paper is organized as follows. In Section II, the nonlinear dynamical model of a wastewater treatment process taking place inside a SBR is presented. In Section III, we address the dynamical adaptive SMC solution for this nonlinear bioprocess. Numerical simulations are included in order to test the behavior and the performance of the proposed control strategy. The results obtained via simulation are presented in Section IV. Finally, Section V concludes the paper.

2. Dynamical Nonlinear Model of Wastewater Treatment Bioprocess

A biotechnological process carried out inside a bioreactor can be defined as a set of $m$ biochemical reactions involving $n$ components (with $n > m$). By using the mass balance of the components and obeying some modeling rules, a dynamical state-space model can be obtained [1], [10]. The model is described by the equations:

$$\dot{\xi} = K \cdot \varphi(\xi) - D\xi + F - Q. \tag{1}$$

In (1), $\xi = [\xi_1, \xi_2, \ldots, \xi_n]^T$ is the vector of instantaneous concentrations (state vector), $\varphi = [\varphi_1, \varphi_2, \ldots, \varphi_n]^T$ is the vector of reaction rates (reaction kinetics), $K = [K_{ij}]$ is the matrix of yield coefficients and $D$ the dilution rate. $F = [F_1, F_2, \ldots, F_n]^T$ is the vector of supply rates, and $Q = [Q_1, Q_2, \ldots, Q_n]^T$ is the vector of removal rates of components in gaseous form.

The nonlinear model (1) describes the behavior of an entire class of bioprocesses and is referred to as the general dynamical state-space model of this class [1]. The process considered next is the treatment of wastewater from chemical industry, containing toxic or recalcitrant compounds (4-chlorophenol 4CP), which takes place inside a SBR located at Universidad Nacional Autonoma de Mexico, Mexico City [4], [5]. The process consists of an aerobic growth reaction and an endogenous respiration one, with the reaction scheme:

$$S + C \rightarrow X, \quad X + C \rightarrow X \tag{2}$$

with $S$, $C$ and $X$ organic matter (substrate), dissolved oxygen and biomass, respectively. A detailed model of this fed-batch process, referred to as EM1, was obtained after a lot of experiments in [4], [5]. From the reaction scheme (2), using the mass balance equations, and assuming that the dissolved oxygen is not limiting and biomass decay is negligible, the next model is obtained [4]:

$$\dot{X} = \mu(S)X - DX, \quad S = -k_1\mu(S)X + DS_{in} - S, \quad \dot{C} = -k_2\mu(S)X - bX + DC_{in} - C + K_e a(C_s - C), \quad \dot{V} = D\dot{V} = F_{in}, \tag{3}$$

where $\mu$, $F_{in}$, $V$, $k_1$, $k_2$, $S_{in}$, $C_{in}$, $b$, $K_e a$ and $C_s$ are: specific growth rate, inlet flow rate, volume, yield coefficients, influent substrate and dissolved oxygen concentrations, endogenous respiration kinetic coefficient, gas-liquid transfer coefficient, and oxygen saturation concentration. $D = F_{in} / V$ is the dilution rate, and $\varphi = \mu(S)X$ the reaction rate.

Considering the first three equations of (3), and denoting: $\xi = [X, S, C]^T$, $F = [0 DS_{in} DC_{in}]^T$, $K = [1 - k_1 - k_2]^T$, $\tilde{Q} = [0 0 bX - K_e a(C_s - C)]^T$ - the extended vector of removal rates, then a model of the form (1) is obtained:
\[ \dot{\xi} = K \cdot \varphi(\xi) - D\xi + F - \bar{Q}. \] (4)

The specific growth rate can be modeled as Haldane law that considers the substrate inhibition on growth [1], [10]:

\[ \mu(S) = \mu^* S / (K_s + S + S^2 / K_i), \] (5)

where \( \mu^* \) is the maximum specific rate, \( K_s \) and \( K_i \) the Michaelis-Menten and inhibition constants.

The strongly nonlinear character of this model is given by the reaction kinetics. In practice, the yield coefficients, the structure and parameters of reaction rates are partially known or even completely unknown.

3. Adaptive SMC Design for the Bioprocess

3.1. Sliding mode control law design

A common problem in the fed-batch bioprocess control is that of generating the substrate feed rate profile to optimize a performance criterion [1]. For the wastewater treatment process, the main objective is to maintain a low level of the pollutant (substrate) concentration.

The control goal is that the substrate concentration \( \xi_2(t) = S(t) \) to track the desired substrate trajectory \( S^*(t) \), with the feeding rate as control action: \( u(t) = F_{in}(t) \). On another hand, the output of the controlled bioprocess will be \( y(t) = S(t) - S^*(t) = \xi_2(t) - \xi_2^* \). It results that the control goal is to force the output to zero.

From the analysis of the nonlinear model (3), it can be obtained that the first two equations are not influenced by the dynamics of the dissolved oxygen concentration. Therefore, the model which will be considered in order to design the adaptive SMC strategy is the next one:

\[ \begin{align*}
\dot{\xi}_1 &= \mu(\xi_2)\xi_1 - \left( u/V \right)\xi_1, \\
\dot{\xi}_2 &= -k_1\mu(\xi_2)\xi_1 + \left( u/V \right)(S_{in} - \xi_2), \\
y &= \xi_2 - \xi_2^*.
\end{align*} \] (6)

The design of the adaptive SMC strategy will be achieved starting with a compact representation of the nonlinear single-input single-output model (6):

\[ \dot{\xi} = f(\xi, u), \quad y = h(\xi) \] (7)

The SMC strategy can be applied for the bioprocess if the so-called observability matrix has full rank [7], [8]:

\[ O = \begin{bmatrix} \frac{\partial h(\xi)}{\partial \xi} & \frac{\partial h^{(1)}(\xi)}{\partial \xi} \end{bmatrix}^T. \] (8)

After straightforward calculation, taking into account the system (6) with the kinetics (5), it can be obtained that the rank is everywhere equals to 2, except on line \( \xi_2 = 0 \), which is devoid of practical significance.

Assume that \( u = \bar{u}, \xi = \bar{\xi}(\bar{u}) \) expresses a constant equilibrium point for (6), such that \( h(\bar{\xi}(\bar{u})) \) is zero. The stability nature of an equilibrium point \( u = \bar{u} \) of the zero dynamics determines the minimum or non-minimum phase character of (6) [7], [8]. Next, the constant equilibrium point is represented as \( (\bar{\xi}(\bar{u}), \bar{u}, \bar{y} = h(\bar{\xi}(\bar{u})) = 0) = (\bar{\xi}(\bar{u}), \bar{u}, 0). \) We will suppose next that the system is locally minimum phase around \( (\bar{\xi}(\bar{u}), \bar{u}, 0). \)

The asymptotic output stabilization problem for (6) is based on the design of a feedback controller. The control \( u \) must to force the system output \( y \) to converge asymptotically to the setpoint value (in our case zero). For control design purposes, consider the following auxiliary output function (the sliding surface) \( \sigma: \mathbb{R}^n \to \mathbb{R}^n \):
The parameter $\gamma_i$ of (9) is real and positive. The idea for the design of SMC law is to use the auxiliary output function (9) $\sigma = 0$ as a sliding surface, and so to force the system trajectories to remain on this surface [3], [7], [8].

**Proposition 1.** The discontinuous feedback control law:

$$
\dot{u}(t) = (V/(S_{in} - \xi_2))(k_1\mu\xi_1 + (u/V) - \gamma_1) \\
(-k_1\mu\xi_1 + (u/V)(S_{in} - \xi_2)) + k_1\mu(-u/V)\xi_1 -
k \text{sgn}[\gamma_1(\xi_2 - \xi^*_2) - k_1\mu\xi_1 + (u/V)(S_{in} - \xi_2)]
$$

(10)

imposes to the output of system (6), in finite time, a linearized dynamics of the form

$$
\sigma(\xi) = \gamma_1(\xi_2^* - \xi_2) - k_1\mu\xi_1 + (u/V)(S_{in} - \xi_2) = 0
$$

(11)

In the expression of the dynamical SMC law (10), $k$ is a strictly positive scalar and $\mu = \mu'(\xi_2) = d\mu(\xi_2)/d\xi_2$.

The proof is direct. If the sliding surface (9) is zeroed by means of a discontinuous control strategy, after some simple but tedious calculations, it follows that the time response of the controlled output $y$ is ideally governed by an asymptotically stable linear time-invariant dynamics.

### 3.2. Adaptive sliding mode control law strategy

When some parameters of system (6) are unknown, then the previous SMC law must be associated with an adaptation law. For example, if there are some uncertainties regarding the yield coefficient $k_1$ and maximum specific growth rate $\mu^*$, then we consider that the vector of unknown parameters is $\theta = [k_1 \, \mu^*]$. The model (6) is rewritten as

$$
\dot{\xi} = f_1(\xi) + f_2(\xi) + g(\xi)u, \quad y = h(\xi),
$$

(12)

with

$$
f_1(\xi) = \begin{bmatrix}
\xi_1^2 \\
K_S + \xi_2 + \xi^*_2 / K_f
\end{bmatrix},
\quad f_2(\xi) = \begin{bmatrix}
-\xi_1 \\
K_S + \xi_2 + \xi^*_2 / K_f
\end{bmatrix},
\quad g(\xi) = \begin{bmatrix}
-\xi_1 \\
1/V (S_{in} - \xi_2)
\end{bmatrix}^T.
$$

From (8) it can be seen that the full rank of observability matrix is preserved. The sliding surface (9) becomes

$$
\sigma(\xi) = \gamma_1(\xi_2^* - \xi_2) - \theta_2\nu\xi_1 + (u/V)(S_{in} - \xi_2)
$$

(13)

with $\nu(\xi_2) = \xi_2 / (K_S + \xi_2 + \xi^*_2 / K_f)$.

Then the dynamical SMC law (10) is rewritten as

$$
\dot{u}(t) = (V/(S_{in} - \xi_2))(\theta_2\nu'\xi_1 + (u/V) - \gamma_1) \\
(-\theta_2\nu\xi_1 + (u/V)(S_{in} - \xi_2)) + \theta_2\nu(\theta_1\nu - (u/V))\xi_1 -
k \text{sgn}[\gamma_1(\xi_2 - \xi^*_2) - \theta_2\nu\xi_1 + (u/V)(S_{in} - \xi_2)]
$$

(14)

with $\nu' = \nu'(\xi_2) = d\nu(\xi_2)/d\xi_2$.

The time derivative of the sliding surface (13) is

$$
\dot{\sigma}(\xi) = -\theta_2\nu'\xi_1 - \theta_2\nu\xi_1 \\
+ \theta_2\nu\xi_1(u/V) + (u/V)(S_{in} - \xi_2) \\
+ (\gamma_1 - (u/V))[-\theta_2\nu\xi_1 + (u/V)(S_{in} - \xi_2)]
$$

(15)

The estimate of the sliding surface (13) is written using the estimates of $\theta$:
\( \hat{\sigma} = \gamma_1(\xi_2 - \xi_3 - \hat{\theta}_2v\xi_1 + (u/V)(S_{in} - \xi_2) ) \)  \hspace{1cm} (16)

If in the SMC law (14) the estimates of unknown parameters are used, the next dynamical adaptive control SMC law is obtained:

\[
\dot{u}(t) = (V/(S_{in} - \xi_2))(\hat{\theta}_2v\xi_1 + (u/V) - \gamma_1).
\]
\[
(-\theta_2v\xi_1 + (u/V)(S_{in} - \xi_2)) + \hat{\theta}_2v(\hat{\theta}_1v - (u/V))\xi_1 - k\text{sgn}[\gamma_1(\xi_2 - \xi_3) - \hat{\theta}_2v\xi_1 + (u/V)(S_{in} - \xi_2)]
\]  \hspace{1cm} (17)

If the control law (17) is used to regulate the evolution of the time derivative \( \hat{\sigma} \), then from (15), (16) we obtain

\[
\dot{\sigma} = -k\text{sgn}(\hat{\sigma}) + (\alpha_1 - \hat{\alpha}_1)(-v^2\xi_1)
+ (\alpha_2 - \hat{\alpha}_2)(-v^2\xi_1(u/V) - v^2\xi_1)
\]  \hspace{1cm} (18)

where \( \alpha_1 = \theta_1, \alpha_2 = \theta_2 \).

The vector \( \alpha = [\alpha_1, \alpha_2]^T \) contains all possible ordered homogeneous multinomial expressions in \( \theta_1, \theta_2 \). By using this overparameterization, the linearity of functions involved in (18) with respect to the parameters is preserved [7], [8]. Let \( \hat{\alpha} \), the estimate of vector \( \alpha \), be defined as \( \hat{\alpha} = [\hat{\alpha}_1, \hat{\alpha}_2]^T \). The relation (18) can be expressed as a linear function of the parameter estimation error \( \hat{\alpha} = \alpha - \hat{\alpha} \):

\[
\dot{\hat{\alpha}} = k\text{sgn}(\hat{\sigma}) + \hat{\alpha}^T R(\xi)
\]  \hspace{1cm} (19)

where \( R \) is the regressor vector:

\[
R = [-v^2\xi_1 - v^2\xi_1(u/V) - v^2\xi_1]^T
\]  \hspace{1cm} (20)

The sliding surface estimation error is denoted \( \hat{\sigma} = \sigma - \hat{\sigma} = \hat{\alpha}^T R_0(\xi) \), with the sliding surface regressor:

\[
R_0 = [0 - v^2\xi_1]^T
\]  \hspace{1cm} (21)

Let us consider the Lyapunov function:

\[
V(\sigma, \hat{\alpha}) = \frac{1}{2}\hat{\sigma}^2 + \frac{1}{2}\hat{\alpha}^T P\hat{\alpha}
\]  \hspace{1cm} (22)

where \( P \) is a known positive matrix. The time derivative of this function along the trajectories of (20) leads for our bioprocess to an adaptation law of the form:

\[
\dot{\hat{\alpha}} = \hat{\alpha} - \hat{\alpha} = -P^{-1} \left[ (\hat{\sigma} + \hat{\alpha}^T R_0) R - kkr_0 \text{sgn}(\hat{\sigma}) \right]
\]  \hspace{1cm} (23)

with \( R, R_0 \) given by (20) and (21). Then we obtain that \( \dot{V}(\sigma, \hat{\alpha}) = -k\|\hat{\sigma}\| \leq 0 \), i.e. the function decreases along the trajectories of the controlled system.

Finally, the adaptive SMC law is obtained from (17), using the estimate of the overparameterized vector \( \hat{\alpha} \). In fact, this adaptive SMC control law consists in the sliding mode controller:

\[
\dot{u}(t) = (V/(S_{in} - \xi_2))(\hat{\alpha}_2v\xi_1 + (u/V) - \gamma_1).
\]
\[
(-\alpha_2v\xi_1 + (u/V)(S_{in} - \xi_22)) + \alpha_2v(\hat{\alpha}_1v - (u/V))\xi_1 - k\text{sgn}[\gamma_1(\xi_2 - \xi_32) - \hat{\alpha}_2v\xi_1 + (u/V)(S_{in} - \xi_22)]
\]  \hspace{1cm} (24)

and the adaptation law (23) written by components as:
\begin{align*}
\dot{\tilde{x}}_1 &= -\frac{1}{p_{11}} (-\sigma \tilde{v}^2_{x_1} + \tilde{a}_2 \sqrt{\xi}^2_{x_1}), \\
\dot{\tilde{x}}_2 &= -\frac{1}{p_{22}} (-\sigma - \tilde{a}_2 \sqrt{\xi}^2_{x_1}) (\tilde{v}^2_{x_1} - \sqrt{\sigma} - \tilde{v}^2_{x_1}) + k \text{sgn}(\tilde{\sigma}) \tag{25}
\end{align*}

with the positive matrix \( P \) chosen as a diagonal matrix \( P = [p_{ii} > 0]_{i=1,2}, p_{ij} = 0, i \neq j \).

4. Simulation Results

The behavior and the performance of the proposed dynamical adaptive SMC strategy were analyzed by performing several simulations. The wastewater treatment bioprocess has been simulated by numerical integration of equations (3), (5), and the next process parameters [4], [5]:

\[
\mu^* = 0.19 \text{ h}^{-1}, K_S = 60 \text{ mg} 4\text{CP}/L, K_I = 3.7 \text{ mg} 4\text{CP}/L, \\
k_I = 3.5 \text{ mg} 4\text{CP}/mg\text{VSS}, k_2 = 1.036 \text{ mg} 4\text{DO}/mg\text{VSS}, \\
b = 0.0059 h^{-1}, K_L a = 16.8 h^{-1}, C_S = 600 \text{mg/L}. 
\]

The simulated control tasks considered the problem of stabilizing the output \( y \) to \( y^* = 0 \), which is equivalent to the substrate concentration converging to a pre-specified constant reference value \( S^* \). In fact, the closed loop system was tested for a step profile of substrate reference, chosen such that a low level of pollutant is obtained. The simulation was conducted in some realistic conditions.

Thus, the bioprocess parameters \( \theta_1, \theta_2 \) were considered unknown and the adaptive SMC law (24), (25) was implemented. Often the on-line measurements of biomass concentration are unavailable. In this case, an asymptotic observer can be used for the reconstruction of \( X \) from the measurements of substrate and dissolved oxygen concentrations [2]. The implemented adaptive SMC law will use the estimations \( \tilde{X} = \tilde{\xi} \) provided by the state observer [2]. Moreover, the measurements of dissolved oxygen concentration (which are used by the asymptotic observer) are considered corrupted with an additive Gaussian noise of 3% from its nominal value. Lastly, another realistic issue considered in the simulation process was a parametric disturbance that occurs in the profile of influent substrate concentration. Thus, \( S_{10} \) was chosen of sinusoidal form (20% around the nominal value of 3500 mg/L).

The control parameters used in (24), (25) are set to \( k = 3, \gamma_1 = 5, p_{11} = 10, p_{22} = 10 \). The choice of elements of matrix \( P \) affects the parameter estimation dynamics. The values of these positive elements are selected via a trial and error process.

Fig. 1 depicts the time evolution of the substrate concentration versus the reference, for both the adaptive SMC and the exact linearizing control law [2], which is a benchmark (i.e. the ideal case, when all parameters and kinetics are known, and all the state variables are on-line available). It can be noticed the good behavior of the adaptive sliding mode controlled system, despite the action of the parametric disturbances and the presence of noisy data. The control input generated by the adaptive SMC exhibits a small chattering. The choice of \( k \) establishes a trade off between tracking precision (a big value for \( k \)) and small chattering (a small value for \( k \)).

![Figure 1](image-url)

**Figure 1.** Time evolution of the substrate concentration versus its reference (the exact linearizing control - benchmark case, and the adaptive SMC case).
Fig. 2. Time profile of the control action.

Fig. 3. The evolution of the biomass concentration and of its estimate.

Fig. 4. Profiles of the sliding surface and of its estimate.

Fig. 3 presents the profiles of the “true” biomass concentration and its estimate provided by the asymptotic observer, slightly influenced by the noisy data of dissolved oxygen concentration used by the asymptotic observer. In Fig. 4 the time profiles of sliding surface and of its estimate are depicted. The estimates of the unknown parameters, which are not presented here for space reason, converge to constant values, but they do not coincide with their “true” values, which is generic for this kind of adaptive algorithm.

5. Conclusion

In this work, a dynamical adaptive sliding mode control was designed for a wastewater treatment bioprocess inside a SBR. The obtained results show that if the sliding surface exhibits an explicit dependence on the uncertain parameters of a bioprocess, an estimate of the switching surface can be used for the
generation of the control action. The estimated trajectory of the sliding surface is driven to zero by means of a Lyapunov function-based SMC strategy.

The obtained results show a good behavior of the controlled wastewater treatment process. The system was tested using realistic simulations, considering parametric disturbances and noisy measurements. The control goal – the maintenance of a low level of pollutant concentration, is achieved despite these disturbances. As expected, the dynamical adaptive SMC ensures a smooth behavior of the output. However, a small chattering of control action is noticed. The choice of $k$ establishes a compromise tracking precision / small chattering.

The results are quite encouraging from a simulation viewpoint and show the robustness of the adaptive SMC and good setpoint regulation performance.

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7. References


