Modeling and Simulation of Correlated K-Distributed Sea Clutter Based on ZMNL

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Abstract. Researches on sea clutter are of great significance for the detection of targets on the sea in complex clutter background. Correlated K-distribution is often offered as a non-Gaussian distribution model which can well describe the characteristics of radar sea clutter. Whether the clutter model is accurate and flexible or not, is an important criterion accessing the performance of radar signal simulator. Researches on modeling and simulation of clutter are helpful for the design of radar signal simulator and radar system. In this paper a method for modeling and simulation of sea clutter is proposed, correlated K-distributed sea clutter is then simulated by employing zero memory non-linearity (ZMNL) transformation based on the autocorrelation functions of K-distribution and Gaussian distribution. Results demonstrate the approach is accurate, effective and feasible.

Keywords: Radar sea clutter; Correlated K-distribution; Zero memory non-linearity; Modeling; Simulation

1. Introduction

When radar is searching and tracking targets on the sea, sea clutter exerts a baneful influence on radar detection and track simulations. Therefore, it is of great significance to simulate sea clutter effectively and accurately for the design of optimal radar signal processor and the simulation of radar system.

There have been extensive experiments on the properties of sea clutter. Based on the data obtained, as for low resolution radar, rayleigh distribution can describe the amplitude probability distribution of sea clutter, when a number of equivalent elementary scatters are contained in each radar range resolution location and subject to the requirements of center limit theorem. On the other hand, as for high resolution radar, the amplitude probability distribution of sea clutter exhibits two main characteristics, one is there is an extended ‘tail’ in the region with a high probability, the other is there is a high ratio of standard deviation - average. At present, Lognormal, Weibull and K-distributions are commonly used for illustrating the non-rayleigh probability distribution functions [1][2].

Jakeman [3] first proposed K-distribution as a model for electromagnetic scattering in 1976, Oliver [4] and Ward [5] presented a model for correlated K-distributed clutter. As for the radar sea clutter in a low incidence angle, K-distribution is commonly used as the amplitude distribution model. Correlative parameters and temporal and spatial correlations are mainly determined by radar scheme and environment parameters. Researches on the envelope model of sea clutter echo acquired from high resolution radar working in a low incidence angle show that, K-distribution can not only matches well with the amplitude distribution of sea clutter data observed within a wide range, but also simulate the interpulse correlation properties of sea clutter echo accurately [6]. Besides, as opposed to other probability models, K-distribution is well constructed in clutter scatter mechanism.
In this paper, we present a method of modeling and simulating sea clutter based on zero memory non-linear (ZMNL) correlated K-distributed clutter, and explain the whole implementation process. Correlated K-distributed sea clutter generated by ZMNL is mainly studied. At last, radar sea clutter is simulated and the results are analyzed. Simulation results demonstrate the validity of the algorithm.

2. Radar Sea Clutter Modeling

The modeling of radar sea clutter mainly include two aspects, one is to ascertain the amplitude distribution and power spectrum of sea clutter, the other is to ascertain the parameters of amplitude distribution and power spectrum model in accordance with concrete radar scheme and working environment.

2.1. Amplitude Distribution Model

Correlated K-distribution provides a wide available range, which is suitable for different types of radar sea clutter. Meanwhile, K-distribution can not only simulate the extended ‘tail’ of amplitude distribution of sea clutter, but also simulate the temporal and spatial correlation properties of sea clutter. According to the correlated K-distributed clutter scatter mechanism, K-distribution model consists of two parts which contain different clutter fluctuations, one is the slow-varying variables based on amplitude modulation, which are often denoted by the Gamma distribution, the other is the fast-varying variables based on speckle distribution determined by the multipath-clutter scatter properties within any range unit, which are often linked by the internal motion of scatters. Therefore, we can regard the correlated K-distribution as a compound Gaussian process, with which the power is modulated by the random process with Gamma distribution. The probability density function is denoted as [6]:

\[ p(\gamma) = K[\gamma; a, v] = \frac{2}{a \Gamma(v+1)} \left( \frac{\gamma}{2a} \right)^{v} K_{v} \left( \frac{\gamma}{a} \right), \gamma > 0, v > -1 \]  

where \( a \) is the scale parameter, which is determined by the average power of clutter, \( v \) is the shape parameter, \( \Gamma(\cdot) \) is the Gamma function, \( K_{v}(\cdot) \) is the modified Bessel function of order \( v \).

2.2. Spectrum Model and Correlative Parameters

Sea clutter fluctuates slowly and has a high pulsed correlation, which is often described by the power spectrum. In the radar sea clutter simulation an instance of models adopted belong to Gaussian distribution or approximate Gaussian distribution. Take Gaussian spectrum for example, it is always defined by the half width of power spectrum, which is denoted as:

\[ S(f) = \exp \left\{ -a((f - f_{0})/f_{3dB})^{2} \right\} \]  

where \( f_{0} \) is the maximum spectrum, \( f_{3dB} \) is the half width of power spectrum determined by the average speed of ocean wave and the radar wavelength, \( a \) is adopted as constant 1.665.

3. Radar Sea Clutter Simulation

Since the amplitude probability distribution and power spectrum model of sea clutter are determined, in order to simulate sea clutter, the first is to generate a group of correlated K-distributed random sequences. Spherically invariant random process (SIRP) [7] and zero memory non-linearity (ZMNL) [8] approaches are commonly used to generate correlated random variables (RVs) with a certain probability distribution. As the SIRP approach is restricted by the sequence orders and autocorrelation function, it is very difficult to form a fast algorithm for large operation. Therefore, ZMNL approach is often used to simulate correlated radar clutter, of which the main idea is to generate a correlated Gaussian random process, and then to generate correlated RVs via non-linear transformation.

3.1. Correlated Gaussian Random Sequence Generation

In order to generate clutter sequences, uncorrelated Gaussian RVs are firstly passed through linear filters to generate correlated Gaussian RVs. Linear filters can be synthesized in time and frequency domain. It is difficult to deal with a long random sequence in the time domain, thereby correlated Gaussian RVs are always generated in the frequency domain. Fig. 1 shows the generation of correlated Gaussian RVs.
As is shown in Fig. 1, correlated coefficient sequence $r(n)$ is first transformed into $R(\omega)$ using FFT, and then the amplitude of linear filter is computed as $|H(\omega)| = \sqrt{R(\omega)/r(0)}$. Meanwhile, uncorrelated Gaussian sequence $x(n)$ is transformed into $X(\omega)$. The spectral density of correlated Gaussian sequence is deserved by a linear product, which is denoted as $S_r(\omega) = |H(\omega)|^2 : X(\omega) = R(\omega)/r(0)$. The correlated Gaussian RVs are finally received by transforming $S_r(\omega)$ into time domain using IFFT, with which the spectral density is $R(\omega)/r(0)$.

### 3.2. Correlated K-distributed Sea Clutter Generation

The power spectrum of sea clutter $z^2$ can be denoted as a product of two RVs $z = XY$, where $X$ submits to speckle distribution, $Y$ submits to Chi distribution.

Fig. 2 shows the generation of K-distributed clutter RVs by employing ZMNL algorithm. The RVs $V_{1,\theta}, V_{2,\theta}, \ldots V_{\theta,2,\theta}$, are independent, uncorrelated and identically distributed. A total of $\theta+2$ RVs are processed as follows: the first $\theta$ comprise the random variable $Y$, and the remaining two comprise the exponentially distributed random variable $X_j$. It is easy to show that the K-distributed probability density function of $Z_j$ is formed from the product of the square roots of $X_j$ and $Y_j$. It is now necessary to define the two filters $H_1(\omega)$ and $H_2(\omega)$ so that $Z_j$ is properly correlated after being formed by the non-linear processing.

$H(\omega)$ is found as follows. First, a realization of $Z_j$ beginning with zero mean, Gaussian distributed RVs is needed. Second, the specified correlation coefficient for $Z_j$, $s_{ij}$ is computed using IFFT for the spectrum of $Z_j$. We define $r_{ij}$ and $q_{ij}$ as the correlation coefficients of the first $\theta$ and the remaining two random variables. In [8], the expression describing the correlation coefficients is denoted as:

$$s_{ij} = \Lambda \left[ F_1(-1/2, -1/2; 1; 1/2, r_{ij}^2) \right]$$

$$\frac{\Gamma(\nu + 3/2) \Gamma(\nu)}{\Gamma(\nu + 1)}$$

with $\Lambda = \frac{\Gamma(\nu + 3/2) \Gamma(\nu)}{\Gamma(\nu + 1)}$, where $F_1(a, b; c; z)$ is the Gaussian hypergeometric function, defined as:

$$F_1(a, b; c; z) = \frac{\Gamma(c)}{\Gamma(a) \Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a + n) \Gamma(b + n)}{\Gamma(c + n) n!} z^n$$

As described above, the probabilities, moments, and correlation coefficients of the joint probability density function for $Z_j$ are functions of two parameters: $r_{ij}$ and $q_{ij}$. Clearly, when $r_{ij}$ and $q_{ij}$ are set in different exponents, the resulting evaluation for the relation between $s_{ij}$ and $r_{ij}$ shows a sharp distinction. The nonlinearity can be decreased to minimum if we can set $r_{ij} = q_{ij}$. That is, to minimize the linearity between the correlated Gaussian-distributed RVs and correlated K-distributed RVs spectrum.

We can see that the model employs $\theta$ random variables $(W_{1,\theta}, W_{2,\theta}, \ldots W_{\theta,\theta})$ to form the Gamma distributed random variable $Y$, which is combined with the exponentially distributed random variable $X_j$ to find the K-distributed $Z_j$ of order $v = \theta/2 - 1$. 

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**Fig.1. Generation of correlated Gaussian random variables**

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**Fig.2. Generation of K-distributed clutter**
Because \( \theta = 1, 2, \ldots \), the parameter \( \nu \) assumes half integer and integer values beginning with \(-1/2\). While the expressions for the probability density function and moments of \( Z \) are correct for any \( \nu \), the generation of correlated, K-distributed random variables is not defined for general \( \nu \). We suspect that the model set forth in Fig. 2 bounds the problem of clutter representation for general \( \nu \). However, [9] reports on a method to extend the results to general \( \nu \) for K-distributed, spatially correlated clutter.

4. Simulation Results and Analysis

As described in Section III, correlated K-distributed random variables can be generated. What should be addressed is that, as Gaussian spectrum is adopted, we often define the sampling interval as \( \Delta f = 2f_{\mu} / N \), where \( N \) is the length of clutter sequence. Generally speaking, \( \Delta f \) is often determined in accordance with radar parameters.

To prove the validity of the method, we can simulate the K-distributed clutter. Simulation parameters are set as follows: half width of power spectrum \( f_{\mu} = 60Hz \), shape parameter \( \nu = 1.5 \), scale parameter \( a = 1 \), sampling numbers \( N = 1024 \). Fig. 3 shows the amplitude statistical distribution of sea clutter and Fig. 4 shows the amplitude probability distribution of sea clutter, from which we can see that the amplitude probability distribution demonstrates a good simulation performance. All is illustrated that K-distribution can simulate the amplitude distribution of sea clutter accurately.

![Amplitude statistical distribution of K-distributed clutter](image)

![Amplitude probability distribution](image)

Clutter temporal correlation is often illustrated by the spectral density. Fig. 5 shows the clutter power spectrum estimation by employing Burg approach, making a comparison with the real power spectrum. As is shown, a high compatibility demonstrates employing K-distribution can effectively simulate the temporal correlation properties of sea clutter. Fig. 6 shows the time-territory waveform of K-distributed clutter.

![Power spectrum](image)

![Time-domain waveform of K-distributed clutter](image)

As described above, we can conclude that the approach of employing ZMNL to generate correlated K-distributed RVs is accurate, and can effectively simulate the amplitude distribution and temporal correlation properties of sea clutter.
5. Conclusion

With the progressing development of modern radar technology, it is increasingly important to model and simulate the radar clutter, which is also the prerequisite for the realization of radar optimum design. Correlated K-distribution is one of the models recognized which can reflect the properties of radar sea clutter. In this paper the correlated K-distribution model is analyzed and established, and then the model is simulated by employing ZMNL approach. A high compatibility with the real results proves the validity of the method. The correlated K-distributed sea clutter can be applied in the design of optimal radar signal processor and the simulation of radar system.

6. References