A VSS-NLMS Algorithm Designed for DTMB On-Channel Repeater

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Abstract. In this paper, we use adaptive filter to realize the echo cancellation strategy in On-Channel Repeater (OCR) based on the National Standard of China for Digital Terrestrial Multimedia Broadcasting (DTMB). In order to solve the conflicting requirement of fast convergence and low misadjustment of the Normalized Least Mean Square (NLMS) algorithm, this paper proposes a Variable Step-Size Normalized Least Mean Square (VSS-NLMS) algorithm. This algorithm does not require any a priori information, so it is easy to control in real world. Simultaneously, the simulation results indicate that this algorithm has a better performance in convergence rate and misadjustment in steady state echo cancellation compared with the conventional LMS algorithm and conventional RLS algorithm.

Keywords: Lms, On-Channel Repeater (Ocr), Dtmb, Echo Cancellation, Adaptive Filter

1. Introduction

On-Channel Repeater (OCR) advantages include better coverage, less interference, less power, and higher reliability. Therefore, OCR gradually used in DTMB as coverage extenders or gap fillers mainly in areas that are shadowed by terrain from the base station can promote the development of the digital television in China.

A particular problem with the use of OCR in DTMB system is the echo interference signal from the transmitting antenna to the receiving one which may cause some negative effects on its performance [1]. The traditional way to prevent the instability and oscillation caused by the echo is to control the forward gain by Automatic Gain Control (AGC) or to separate the transmitting and receiving antennas physically which can increase the isolation between the transmitting and receiving antennas, but the coverage area will be reduced and the installation cost will be increased [2].

After utilizing bandpass filter in Radio Frequency (RF) and Intermediate Frequency (IF) for implementing the echo cancellation part of OCR, the method to fulfill echo cancellation in baseband has been generated [3], and LMS algorithm is used to cancel the echo due to the fact that it is robust and easy to implement, but the performance of this algorithm, in terms of convergence rate, misadjustment, and stability, is governed by the step-size parameter. Within the stability conditions, it is well known that the choice of this parameter, reflects a tradeoff between fast convergence and good tracking ability on the one hand and low misadjustment on the other hand. To meet this conflicting requirement, the step-size needs to be controlled.

As we all know, the “ideal” algorithm should have a high convergence rate and good tracking capabilities but achieving low misadjustment. To meet these requirements, this article introduces a novel VSS-NLMS algorithm derived from the power of system noise. And it can be seen that the proposed algorithm performs much better than NLMS, with fast convergence and low misadjustment from the simulation results.

This paper is organized as follows. Section 2 presents the basic algorithm model of the OCR with echo cancellation based on conventional LMS; section 3 derives the proposed VSS-NLMS algorithm and its application in cancelling echo in the OCR; section 4 shows the simulation results; section 5 summarizes the main conclusion.

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2. The Mode of OCR with Echo Cancellation

![Diagram of OCR with echo cancellation](image)

Figure 1 The baseband model of the OCR with echo cancellation

Fig. 1 shows the baseband model of the OCR with echo cancellation. The signal on the baseband receiving end is

\[ d(n) = r(n) + y(n) + v(n) \]  

(1)

where \( r(n) \) is the received signal from the base station, and \( v(n) \) is the additive white noise. \( G \) represents the gain of the OCR subtracting the isolation between transmitting and receiving antenna, and \( y(n) \) is the echo interference signal.

\[ y(n) = h^H e(n - D) \]  

(2)

where \( h = [h_0 \ h_1 \ \cdots h_{N-1}]^T \) is the echo coupling channel vector including \( G \); \( e(n) = [e(n) \ e(n-1) \ \cdots e(n-N+1)]^T \) is error signal, which is actually the baseband output signal of OCR; \( D \) is the circuit delay outside the baseband system.

The Adaptive Filter part fulfills the function of cancelling echo based on various criteria. Considering that \( r(n) \) is uncorrelated with \( e(n) \), when the estimated channel is same as the practical channel, MSE reaches the minimum value. As a result, the conventional fix step size LMS algorithm ((4)- (6)) is able to make \( e(n) \) satisfy (3).

\[ E[|e(n)|^2] \approx E[|r(n)|^2] + E[|v(n)|^2] \]  

(3)

\[ \hat{y}(n) = \hat{h}^H(n-1)x(n) \]  

(4)

\[ e(n) = d(n) - \hat{y}(n) \]  

(5)

\[ \hat{h}(n) = \hat{h}(n-1) + \mu(n)x(n)e^*(n) \]  

(6)

where \( \hat{h}(n) = [\hat{h}_0(n) \ \hat{h}_1(n) \ \cdots \ \hat{h}_{N-1}(n)] \) is the estimate value of \( h \), and \( \hat{y}(n) \) is the estimate value of \( y(n) \). \( x(n) \) is the input vector of the transversal filter, which is \( e(n-D) \). \( N \) is the number of taps of the transversal filter and \( \mu(n) \) is a positive scalar known as step-size, which is a constant in the conventional fix step size LMS algorithm.

3. Proposed VSS-NLMS Algorithm Echo Cancellation

We define the \textit{a priori} and \textit{a posteriori} error signals respectively

\[ e(n) = d(n) - \hat{h}^H(n-1)x(n) = r(n) + v(n) + [\hat{h}^H - \hat{h}^H(n-1)]x(n) \]  

(7)
\[ e(n) = d(n) - \hat{h}^H(n)x(n) = r(n) + v(n) + \left[ h^H - \hat{h}^H(n) \right] x(n) \]  

(8)

where \( \hat{h}(n-1) \) and \( \hat{h}(n) \) are estimates of the system \( h \) at time \( n-1 \) and \( n \) respectively. Taking (6)-(8) into account, the relation between the a priori and a posteriori error signals is

\[ e(n) = e(n) \left[ 1 - \mu(n)x^H(n)x(n) \right] \]  

(9)

In order to derive a step-size parameter within the stability conditions, several approaches can be chosen. One reasonable way is to impose that \( e(n) = 0 \), assuming that \( e(n) \neq 0, \forall n \) [4]. Consequently, \( \mu_{NLMS}(n) = \left[ x^H(n)x(n) \right]^{-1} \), which is the classical NLMS algorithm [5], [6]

\[ \hat{h}(n) = \hat{h}(n-1) + x(n) / x^H(n)x(n)e^*(n) \]  

(10)

It can be noted that the above approach holds in the absence of noise and \( r(n) = 0 \). In practice, a positive adaptation constant (usually smaller than 1) multiplies this step size to achieve a proper compromise between the convergence rate and the misadjustment. In order to cancel the a posteriori error in the presence of noise and \( r(n) \neq 0 \), it results from (8) that

\[ \left[ h^H - \hat{h}^H(n) \right] x(n) = r(n) - v(n) \neq 0 \]  

(11)

which will bias the adaptive filter estimate. In this case, the requirement \( \left[ h^H - \hat{h}^H(n) \right] x(n) = 0, \forall n \), implies that \( e(n) = r(n) + v(n) \). Hence, in the new proposed procedure, we wish to find the step-size parameter \( \mu(n) \) in such a way that

\[ E\left[ e^2(n) \right] = \sigma_e^2, \forall n \]  

(12)

where \( E\{ \cdot \} \) denotes mathematical expectation, \( \sigma_e^2 = E\{ [r(n) + v(n)]^2 \} \) is the power of the sum of received signal and system noise. Using the approximation \( x^H(n)x(n) = N \sigma_e^2 = N E\{ x^2(n) \} \) for \( N \gg 1 \), where \( \sigma_e^2 \) is the power of input signal. Compute the mathematical expectation of (9) and square it simultaneously

\[ E\{ e^2(n) \} = \left[ 1 - \mu(n)N\sigma_e^2 \right] \sigma_e^2(n) = \sigma_e^2 \]  

(13)

where \( \sigma_e^2 = E\{ e^2(n) \} \) is the power of the error signal. Developing (13), we can obtain a quadratic equation

\[ \mu^2(n) - \frac{2}{N\sigma_e^2} \mu(n) + \frac{1}{\left( \frac{1}{N\sigma_e^2} \right)} \left[ 1 - \frac{\sigma_e^2}{\sigma_e^2(n)} \right] = 0 \]  

(14)

By solving the quadratic (14), two solutions are obtained, i.e.,

\[ \mu(n) = \frac{1}{x^H(n)x(n)} \left[ 1 \pm \frac{\sigma_e}{\sigma_e(n)} \right] \]  

(15)

Following the analysis from the reference [7], which states that a value of the step-size between 0 and 1 is the preferable over the one between 1 and 2 (even if both solutions are stable, but the former has less steady-state mean square error with the same convergence speed), although that conclusion is based on the affine projection algorithm, it is available for LMS algorithm. Hence, the reasonable solution is

\[ \mu_{Pro-VSS}(n) = \frac{1}{x^H(n)x(n)} \left[ 1 - \frac{\sigma_e}{\sigma_e(n)} \right] \]  

(16)

where \( \alpha(n) = \frac{\sigma_e}{\sigma_e(n)} \) is the normalized step size. Therefore, the proposed VSS-NLMS algorithm is

\[ \hat{h}(n) = \hat{h}(n-1) + \mu_{Pro-VSS}(n)x(n)e^*(n) \]  

(17)

where \( \mu_{Pro-VSS}(n) \) is defined in (16).
We see from (16) that before the algorithm converges, \( \sigma_e(n) \) is large compared to \( \sigma_n \); so \( \mu_{\text{Pro-VSS}}(n) \approx \mu_{\text{NLMS}}(n) \). On the other hand, when the algorithm starts to converge to the true solution, \( \sigma_e(n) \approx \sigma_n \) and \( \mu_{\text{Pro-VSS}}(n) \approx 0 \). This is exactly what we desire to have: both good convergence and low misadjustment. As we can notice, this approach was derived with almost no assumptions compared to all other algorithms belonging to the same family, so it is easy to control in the real world.

In practice, all adaptive algorithms need to be regularized in order to avoid divisions by small numbers. This implies that a positive constant \( \delta \) needs to be added to the denominator of both step sizes \( \mu_{\text{Pro-VSS}}(n) \) and \( \mu_{\text{NLMS}}(n) \).

It is clear that \( \sigma_e(n) \geq \sigma_n \), which implies that \( \mu_{\text{Pro-VSS}}(n) \geq 0 \). In practice, the quantity \( \sigma_e^2(n) \) is estimated as follows:

\[
\hat{\sigma}_e^2(n) = \lambda \hat{\sigma}_e^2(n-1) + (1-\lambda)e^2(n)
\]

where \( \lambda \) is a weighting factor chosen as \( \lambda = 1 - 1/(KN) \), with \( K > 1 \); the initial value is \( \hat{\sigma}_e^2(0) = 0 \).

4. Experiment Result

Reference [1] and [8] prevent system from oscillation by controlling the gain of OCR during the initialization, which set the gain at a low level at first then increase the gain gradually until the aimed gain is reached. However, the output of the baseband is limited by the data width, which cannot increase to the infinity in the practical system. If adjust the RF modules to make sure that the power of the echo signal is in the input range of the receiving end when the output reaches the maximum, the system will have less probability to be disturbed by oscillation.

During the simulation, we add the limiter block to the output of the system in order to limit the output in the range that can be expressed by the practical data. The function of the limiter is as (19), which only consider the real signal. Because we normalize the input signal DTTB and the error signal after echo cancellation is the estimate of DTTB, the real part and imaginary part of error signal must be between -1 and 1.

\[
\text{Limiter}(e) = \begin{cases} 
1 & e > 1 \\
-1 & e < -1 \\
e & \text{otherwise}
\end{cases}
\]

(19)

The test signal used in this simulation is ten DTV signal frames based on DTMB, whose bandwidth is 7.56MHz, the frame head mode of which is PN420. Moreover, the sampling frequency is 30MHz. We choose COST207 RA6 [9] as the echo coupling channel model, the parameters of which is shown in Table I and the Doppler frequency shift is 0Hz. Therefore, we set the order of the echo coupling channel as 16, then assume the gain of OCR is 70dB and the isolation between transmitting and receiving antenna is 55dB.

The experiment investigates the conventional fix step size LMS (Con-LMS), conventional normalized LMS (Con-NLMS), and conventional recursive least squares (Con-RLS), proposed variable step size NLMS (Pro-VSS-NLMS). The parameters of these algorithms are shown in Table II.

<table>
<thead>
<tr>
<th>Algorithm Type</th>
<th>Parameters</th>
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<tbody>
<tr>
<td>Con-LMS</td>
<td>( \mu = 2^3 )</td>
</tr>
<tr>
<td>Con-NLMS</td>
<td>( \mu = 2^2 )</td>
</tr>
<tr>
<td>Con-RLS</td>
<td>( \delta = 0.002 ) (forgetting factor)</td>
</tr>
<tr>
<td>Con-VSS-NLMS</td>
<td>( \delta = 0.02 )</td>
</tr>
<tr>
<td>Pro-VSS-NLMS</td>
<td>( K = 2 )</td>
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</table>

We use the convergence of the normalized misalignment (in dB), \( 20\log_{10}\left( \frac{\|h - \hat{h}(n)\|}{\|h\|} \right) \), for all the algorithm as a measure of performance.
Fig. 3 shows the misalignment of the system during the initialization in the SNR of 30dB, from which we know that the convergence characteristic of Pro-VSS-NLMS is better than that of Con-LMS, Con-NLMS and RLS.

In addition, Echo Return Loss Enhancement (ERLE) can evaluate the echo cancellation performance in the steady state [10]. ERLE is defined as (20), where \( d_{\text{echo}}(n) \) is the echo received that is similar with the received signal at the receiving end, while \( r_{\text{echo}}(n) \) is the residual echo which is approximate to the error output.

\[
ERLE = 10 \log_{10} \left( \frac{E[d_{\text{echo}}(n)^2]}{E[r_{\text{echo}}(n)^2]} \right) \approx 10 \log_{10} \left( \frac{1}{P} \sum_{n=1}^{P} |d(n)|^2 \right)
\]

From Fig. 3, we know that the echo cancellation performance in steady state of Pro-VSS-NLMS is better than other three algorithms.

5. Conclusion

It is well known that the fast convergence rate and low misadjustment are difficult to satisfy simultaneously in the conventional LMS algorithm. To achieve this conflicting requirement, this article introduces a VSS-NLMS algorithm used in OCR with echo cancellation in DTMB system, which is easy to
control in real world because it does not need any a priori information. Through the simulation results, we notice that the proposed algorithm performs better than conventional fix step size LMS algorithm in the convergence rate and the misadjustment in steady state echo cancellation.

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7. References