An optimal Scheme Based on Local Query for Computer Graphics

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Abstract. The construction of data structure plays a key role for the speedup processing of data in CAD/CAM systems. This paper proposes the schemata whose amount of storage is linearly proportional to the number of entity and implements the retrieval to data in local range. The interrogation of special relations and properties of three dimensional objects are often involved in applying for CAD/CAM, especially in solid modeling. The interrogation and query to data are discussed in this paper in terms of the primitives that we call topological query. The access schemata for three dimensional topological data structure founded on faces, edges and vertexes in graph. Boundary-based half-symmetry, loop and symmetry data structure schemata given in this paper are depended on the primitives of nine kinds of topology queries for creating new entity, and its analysis of storage and time complexity is shown. The time complexity based on the nine kinds of access primitives is confined in local or constant time, so that the optimum schemata for local queries can be implemented in CAD/CAM better.

Keywords: Component; Computer Graphics; Topological Query; entity modelling; CAD/CAM

1. Introduction

Many methods and algorithms for CAD/CAM gave being discussed and developed [1-5]. In solid modeling, new entities can be modified or generated by Euler or Euler-like operations. For data retrieval, the following query may be necessary: “given an entity \( X_i \), find all entities \( Y_j \) connected or adjacent to it”. Queries in the foregoing scenarios are fairly common and generally present, but they depend highly on the interrelation among space objects in time. In this correspondence, we show that this type of queries can be expressed in terms of nine access primitives, and propose optimization schemata in correspond with different storage cost according to boundary-based topology structure

2. Relations, Storage and Time

Suppose the topological entities are \( F(\text{Face}), E(\text{Edge}) \) and \( V(\text{Vertex}) \). To facilitate discussion, we use the following symbols:

\[
\begin{align*}
X_i & \quad \text{— an entity of X type} \\
\{X\} & \quad \text{— the entity set of X type} \\
x & \quad \text{— the entity number of X type} \\
YX_i & \quad \text{— the entity number of Y type connected to X type} \\
X_i(Y) & \quad \text{— the Y set for X} \\
\end{align*}
\]

where \( X, Y = F/E/V \), and the followings are the same except explanation.
Figure 1. a cube example (a) a topology expression of (a), (b) some adjacent relations and (c) nine relations
A boundary data structure can be thought of as a set of adjacent relationships among topological entities. Let a relation be denoted by \(\{X\} \rightarrow \{Y\}\), unless ambiguity arises, the followings are denoted by \(X \rightarrow Y\) for simplicity.

### 2.1 Definition
If there exists \(X \rightarrow Y\), the query of complete relation is
\[
X \rightarrow \{W \subseteq Y \mid (\forall X_i)(X_i \in X) \land (W = YX_i)\}
\]

The relation \(V \rightarrow E\), for example, shown in the Fig. 1, stores its three edges for each vertex in \(V\). Thus for each \(V_i\), its adjacency edges can be modified or accessed. The storage complexity of a relation \(X \rightarrow Y\) can be computed by taking the Sum \(\sum YX_i\). For example, the total storage for \(E \rightarrow V\) is \(\sum VE_i\).

### 2.2 Topological query
Since there nine topological relations formed from \(F\), \(E\) and \(V\), there exist nine data structure access primitives. We will refer to them as topological queries \(T_i\) for \(i = 1, \ldots, 9\) (see Table I). Hence, the time complexity of data structure schemata can be cost by these nine access primitives. \(T_1——T_9:\)

### Table I Table Type Styles

<table>
<thead>
<tr>
<th>Nine access primitives</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_1(V \rightarrow Y))</td>
<td>Given a vertex, find all vertexes connected to it</td>
</tr>
<tr>
<td>(T_2(V \rightarrow E))</td>
<td>Given a vertex, find all edges connected to it</td>
</tr>
<tr>
<td>(T_3(V \rightarrow F))</td>
<td>Given a vertex, find all faces around it</td>
</tr>
<tr>
<td>(T_4(E \rightarrow V))</td>
<td>Given an edge, find the two vertexes connected to it</td>
</tr>
<tr>
<td>(T_5(E \rightarrow E))</td>
<td>Given an edge, find the four edges connected to it</td>
</tr>
<tr>
<td>(T_6(E \rightarrow F))</td>
<td>Given an edge, find the two faces intersecting at it</td>
</tr>
<tr>
<td>(T_7(F \rightarrow V))</td>
<td>Given a face, find all vertexes around it</td>
</tr>
<tr>
<td>(T_8(F \rightarrow E))</td>
<td>Given a face, find all edges bounding it</td>
</tr>
<tr>
<td>(T_9(F \rightarrow F))</td>
<td>Given a face, find all faces around it</td>
</tr>
</tbody>
</table>
We show the example of Fig. 1 based on these nine primitives. Queries T1——T9, correspond to the time complexity measures for direct, indirect, and inverse relations V→V,…,F→F.

3. Bounds for Storage and Time Complexity

This section introduces the techniques for counting storage and for evaluating the time required for answering T1——T9, queries and show the lower and the upper bound for both storage and time for all data structures. For measuring the number of storage locations needed, we will use total number of edges E as the unit.

It is clear that $C^{o}_{2}$ and $C^{o}_{9}$ are bound for storage and time among eight classes of data structures $C^{o}_{m}$ (m=2,…,9) formed from F,E and V. If there exists a direct relation Ti of X→Y for any data structure with $C^{o}_{m}$, Ti is the query of constant time K. If there exist relations E→V and E→F, since an edge is shared by two faces and two vertexes, their storage complexity is the same 2E.

3.1 Lemma

The storage complexity of any relation X→Y except E→E is 2E.

Proof: In the closed solid modeling, since there exists YXi = ZXi = XXi(Xxi = EEi), the time complexity can be evaluated by YXi for any relation X→Z. If each entity Xi is adjacent to YXi edges, and Vi, VjXi(Y)∩Xj(Y)=Yi for any Yi, each edge Yi is stored exactly twice. For E edges, we get $\sum YXi = 2E$.

![Diagram](a) E (b) E (c) F

Storage Time: 4E 7E+2K 6E+EVi+2K 4E 6EVi+3K

Figure 2. The $C^{o}_{2}$ class and winged-edge schema

<table>
<thead>
<tr>
<th></th>
<th>$C^{o}_{2}$</th>
<th>$C^{o}_{3}$</th>
<th>$C^{o}_{4}$</th>
<th>$C^{o}_{5}$</th>
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<tr>
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<td>Best</td>
<td>6E</td>
<td>4EVi</td>
<td>5EVi</td>
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<tr>
<td></td>
<td>worst</td>
<td>+EVi+2K</td>
<td>+2EVi+3K</td>
<td>+4K+5K</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7E+2K</td>
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<table>
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<tr>
<th></th>
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<th>$C^{o}_{7}$</th>
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<tbody>
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<td></td>
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<td>EVi</td>
<td>9K</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td>3E+6K</td>
<td>2E+7K</td>
<td>E+8K</td>
<td>9K</td>
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<table>
<thead>
<tr>
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<th>storage</th>
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<th>14E</th>
<th>16E</th>
<th>20E</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>worst</td>
<td>14E</td>
<td>16E</td>
<td>18E</td>
<td>20E</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3. The storage and time for $C^{o}_{m}$

Fig. 3 shows the storage for any $C^{o}_{m}$ and the upper and the lower bound for both storage and time——(20E, 4E), (7E+2K, 9K).

3.2 Wing-edge structure
Baumgart[4] discovered the use of a fractional relation $F \rightarrow^1 E$ and $V \rightarrow^1 E$ in designing of the wing-edge data structure. $F \rightarrow^1 E$ gives rise to a storage cost of $F$. Similarly, $V \rightarrow^1 E$ costs $V$ in storage. The total storage is therefore $8E+(V+F)=9E$.

For time complexity, there are clearly three constant time queries. For the other six queries, local clocking around a face or around a vertex is required, costing $E_Fi$ or $E_Vi$ in time respectively. Since $E_Vi$ is of the same order as $E_Fi$, we chose $E_Vi$ as the unit. Thus, the total time complexity of the winged-edge is $6E_Vi+3K$.

4. Transmission, Loop, and Symmetry Data Structure and their Optimality

Fig. 2(a) shows the lower and upper bound for storage and time for the class of $C_{2}^{9}$. But if we transform $E \rightarrow F$ into $F \rightarrow E$ then the other schema of $C_{2}^{9}$ class is formed. Storage cost does not change, but since there is the side benefit of a indirect relation $F \rightarrow V$ which can be derived from $F \rightarrow E$ and $E \rightarrow V$, the time complexity decreases.

From a given $Fi$, $EFi$ edges are returned in constant time. For each of $EFi$ edges, exactly two vertexes are returned by $E \rightarrow V$ in constant time, as for $EFi$ edges, the overall time for answering $T7$ is $E_Fi$ or $E_Vi$. which forms a transmit relation schema. It changes global $E$ query into local query $E_Vi$, and also decreases from $7E+2K$ to $6E+E_Vi2K$.

![Figure 4](image)

**Figure 4.** The $C_{3}^{9}$ class (a) half-symmetry (b)loop (c) and (d) others

Suppose we add a new relation $V \rightarrow E$ to the $C_{2}^{9}$ data structure as shown in Fig. 2(a) in terms of the transferability. The additional storage cost is $2E$. There are clearly three constant time queries, but because of increasing $V \rightarrow E$, it gives rise to the side benefit of three indirect queries $V \rightarrow E$, $V \rightarrow V$ and $E \rightarrow E$. Thus, overall time complexity highly decreases to $3E+3E_Vi+3K$.

If we apply the transferability of relations on Fig. 2(b), adding a new relation $V \rightarrow F$, the Fig. 4(b) schema can be brought about. Clearly, all relations can be derived locally, except the three direct relations. It is the addition of the $V \rightarrow F$ that brings about the five side benefit from indirect query which changes from global $E$ to local $E_Vi$.

We add a new relation $V \rightarrow F$ to the $C_{2}^{9}$ data structure as shown in Fig. 2(b), which storage is still $2E$, but the time complexity changes greatly. Hence, the topological data structure independent of global $E$ is raised, the worst time is the local $O (EV2)$. $T_i$, for example, goes through two variable set $Vi(F)$ and $Fi (E)$, so $2FVi\times E Fi$ is required. Because of $FVi=E Vi$, its time is $EV2$. Thus, the overall time complexity for Fig. 4(b) is the $4EV2+2EVi+3K$.

Though the loop relation schema rises the twice order complexity $EV2$, because of the $Y$ adjacent objects of $X$ entity $YXi\leq 6$, the Fig. 4(b) is preferable to (a) and its complexity is proportional to $E$ when $E$ is greater.

4.1 Theorem 1

In the $C_{3}^{9}$ class, the loop relation schema gets the optimum time.
Proof: The first case: if there exists the relation $X \rightarrow X$ (see Fig. 4(c)) in the $C^3$ class ($X=F/E/V$), it only contributes a $K$ constant time and does not raise indirect relations, so no side benefit on time improvement. The best case is the addition of $X \rightarrow X$ on Fig. 2(a), its time complexity is more than that of Fig. 4(a) and (b).

The Second case: if there not exists relation $X \rightarrow X$ in the $C^3$ class, it must be the Fig. 4(d)-like schema. Because of corresponding to the addition of a $K$ constant time in $C^3$ class, its time complexity is greater than that of Fig. 4(a) and (b).

We have shown a loop relation schema of the $C^3$ classes, but it has still the twice order complexity of the local query. Suppose transferability is applied again on the Fig. 4(a), there would be the Fig. 6(e)-like three schemata. For the addition of a relation $X \rightarrow Y$, since there exists $Y \rightarrow X$, $X \rightarrow X$ and $Y \rightarrow Y$ can be answered in linear time $EV_i$, but $Z \rightarrow Z$ is still loop path ($a EV_2$ complexity).

Look at the Fig. 4(a), if a relation $F \rightarrow E$ whose storage is still $2E$ is added again, three $EV_i$ time queries, which can be derived from $X \rightarrow Y/X \rightarrow Z$ and $Y \rightarrow X/Z \rightarrow X$ for any relation $X \rightarrow X$, would be bought about (see Fig. 4(a)schema based on the edge-symmetry).

There are symmetric schemata edge-symmetry, vertex-symmetry and face-symmetry which is similar Fig. 5(b). For vertex-symmetry/face-symmetry, there exists still the twice order complexity from local queries (face/vertex), so that it can be proved that the edge-symmetry is preferable to the others, and also not faster optimal schemata than the Fig. 5(a) in $C^4$.

4.2 Theorem 2

In the $C^4$ class, the edge-symmetry schema gets the optimum time.

![Diagram of schemata](image)

Figure 5. The $C^4$ class (a) edge-symmetry (b) vertex-symmetry (c) winged-edge (d) and (e) others
Proof: The first case: if there exists the relation $X \rightarrow X$ in the $C_4^9$ class, it only contributes a K constant time and does not raises indirect relations. The best case is the addition of $X \rightarrow X$ in $C_3^9$ class which improves the complexity from global E to K. It is more than that of edge-symmetry (see Fig. 5(a) and (d)).

The second case: If it is not a symmetric schema in $C_4^9$ class, and not exists the relation $X \rightarrow X$, then it must be Fig. 5(e) schema which exists twice order complexity (EV2), so that its time is greater than that of the edge-symmetry schema.

It can be seen from Fig. 6 that the left bound curve of the shadow area is the curve to form the optimum schemata with different storage, the right (dash line) is the worst expression on time, and the shadow area is where possible various schemata appear.

5. Conclusion

The optimum schemata for local query have are proposed above, and proved their uniqueness and effective method for constructing boundary data structure in this paper. The idea $C_5^9, C_6^9, C_7^9, C_8^9$ and $C_9^9$ schemata will be formed on the $C_4^9$ symmetric schema, but adding 2E storage only gets the linear change (see the Fig. 6) from EVi to K. It does not raise more optimum schemata (refer to the theorem 1 and theorem 2). The edge-symmetry schema contributes good traversal for searing various objects in 3D entity modeling, and its storage and time cost is preferable to the others schemata.

6. Acknowledgment

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7. References


