Passive Velocity Field Control for AUV Planar Trajectory Tracking

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Abstract. – In order to minimize cost of inspection for a small battle ship’s surface, an Autonomous Underwater Vehicle (AUV) was proposed. However, it is very difficult to achieve both time and path tracking for inspection. Passivity and robustness properties of the Passive velocity field control (PVFC) are required for control the AUV in operating environment. This paper we applied PVFC with dynamics system of the AUV and simulated on circular path tracking. When we tune fine gains, the AUV can be converge to desired path even though the AUV located everywhere on the space as results of simulation are presented.

Keywords: PVFC, AUV, underwater vehicle, planar trajectory tracking

1. Introduction

For preventive maintenance in small battle ship’s surface, we need to put the ship into a dry-doc. Other than expensive operation, there are also risks involved. In order to minimize operating cost and risks, we proposed underwater inspection using a small underwater vehicle (UV) with camera as an inspector. However, it is almost impossible to cover a ship’s surface with human as an operator, due to incapability of human to control an UV to track any given path all the time. Therefore, an autonomous underwater vehicle (AUV) with camera was proposed. However, most control strategies presented depend on time-tracking as well as trajectory tracking which may cause instabilities due to operating disturbances and uncertainties. This paper objective is to apply Passive velocity field control (PVFC) with the AUV for survey mission. PVFC was developed by P. Y. Li in [1], [2]. It is suitable for fully actuated mechanical system which have certainly coordinate and time invariant trajectory. Principles of PVFC are base on coding in term of velocity field on desired path and designing feedback controller to absorb energy when the robot was interacted from physical environment. Afterward he applied PVFC with Self-Pacing for complicated desired path system. Suspension technique was also used to define a velocity field for improve tracking performance [3]. PVFC was combined with integral force controller to enhance robustness system [4]. M. Yamakita applied a decentralized PVFC with multiple robotic (3-wheeled mobile robots) system which was linked them together by rigid bar. These systems were tracking on apart of circular trajectory [5], [6]. T. Narikiyo applied a new formulation of passive velocity field control to control under actuated mechanical systems (UAMS) for under actuated planar three-link manipulator and under actuated planar body [7]. Then, the UAMS was applied with snake board. Simulation results illustrate of the validity of control methodology [8].

2. Passive Velocity Field Control (PVFC)

We consider a general mechanical system with n-dimensional, that dynamics equation can be written by:

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} = G(q)u \]  (1)
Where \( q \) is local coordinates of the robot, \( q = [q_1 \ q_2 \ ... \ q_n]^T \). \( M(q) \) and \( C(q, \dot{q}) \) are the inertia matrix and the Coriolis terms. \( u \) is the coupling control torque, and \( G(q) \) is the matrix relation of each actuator coupling torque.

### 2.1. The Augmented Mechanical System

For PVFC system, the kinetic energy of the closed-loop system is stored in term of the fictitious flywheel dynamics that is one part of dynamics controller.

\[
M_F \dot{q}_{n+1} = \tau_{n+1} \quad (2)
\]

The flywheel inertia \( M_F > 0 \), \( \tau_{n+1} \) is the input torque to the flywheel.

From equation (1) we can rewrite to the dynamics of the augmented mechanical system as

\[
\ddot{q} + \dot{C}(\dot{q}, \ddot{q}) = \dot{G}(\dot{q})\ddot{u} \quad (3)
\]

\[
\ddot{q} = \begin{bmatrix} \dot{M}(q) & 0 \\ 0 & M_F \end{bmatrix} \ddot{\dot{q}} + \begin{bmatrix} C(q, \dot{q}) \\ 0 \end{bmatrix}, \quad \ddot{u} = \begin{bmatrix} u \\ u_{n+1} \end{bmatrix}, \quad \dot{G}(\dot{q}) = \begin{bmatrix} G(q) \\ 0 \end{bmatrix}, \quad \ddot{q} = \begin{bmatrix} q \\ q_{n+1} \end{bmatrix},
\]

where \( \ddot{M}(\ddot{q}) \) is the inertia matrix of the augmented system, \( \dot{C}(\dot{q}, \ddot{q}) \) is the augmented Coriolis matrix, \( \ddot{u} \) is the coupling torque of augmented system, \( \dot{G}(\dot{q}) \) is the matrix relation of each coupling torque of augmented system, and \( \ddot{q} \) is local coordinates of augmented system.

We define the kinetic energy of the augmented dynamic system expressed in term of local coordinates as.

\[
\ddot{q} = \begin{bmatrix} \alpha V(q) \\ V_{n+1}(q) \end{bmatrix} \quad (4)
\]

Total kinetic energy of the augmented system can be written in term of velocity field

\[
\ddot{\dot{V}}(\ddot{q}) = \begin{bmatrix} \ddot{V}(\ddot{q}) \\ \ddot{V}_{n+1}(q) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \dot{M}(\ddot{q}) \ddot{V}(\ddot{q}) + \frac{1}{2} M(q) V^2(q) + \frac{1}{2} M_F V^2_{n+1}(q) \end{bmatrix} \quad (5)
\]

The conservation of energy at constant \( \ddot{E} \)

\[
\ddot{E} = \frac{1}{2} \dot{V}(\ddot{q})^T \ddot{M}(\ddot{q}) \ddot{V}(\ddot{q}) = \frac{1}{2} M(q) V^2(q) + \frac{1}{2} M_F V^2_{n+1}(q) \quad (6)
\]

\[
V_{n+1}(q) = \sqrt{\frac{2}{M_F}(\ddot{E} - \frac{1}{2} V(q)^T M(q) V(q))} \quad (7)
\]

### 2.2. The augmented desired velocity field \( \ddot{V}(\ddot{q}) \)

The desired velocity field \( \ddot{V} \) at every point \( q \) in robot’s configuration space

\[
\ddot{V}(\ddot{q}) = \begin{bmatrix} \alpha V(q) \\ V_{n+1}(q) \end{bmatrix} \quad (8)
\]

\[
\alpha V = \begin{bmatrix} \alpha_x \ddot{v}_d \ddot{x} \\ \alpha_y \ddot{v}_d \ddot{y} \end{bmatrix}
\]

where

\[
\alpha_x = K_x (x_d - x), \quad V_x = \alpha_x v_x
\]

\[
\alpha_y = K_y (y_d - y), \quad V_y = \alpha_y v_y
\]

\( \alpha_x \) and \( \alpha_y \) are the error gains for trajectory tracking

### 2.3. Coupling Control Law

Due to the coupling control torque of the PVFC, it can be written as equation (9).

\[
\ddot{\tau}(\ddot{q}, \ddot{q}) = \ddot{\tau}_{c}(\ddot{q}, \ddot{q}) + \ddot{\tau}_{f}(\ddot{q}, \ddot{q}) \quad (9)
\]

where

\[
\ddot{\tau}_{c}(\ddot{q}, \ddot{q}) = \frac{1}{2E} (\ddot{\omega} \dddot{\ddot{p}}^T - \dddot{\dddot{p}}^T) \ddot{q} \quad (10)
\]

\[
\ddot{\tau}_{f}(\ddot{q}, \ddot{q}) = \gamma (\dddot{\ddot{p}}^T - \dddot{\dddot{p}}^T) \ddot{q} \quad (11)
\]

\[
\ddot{p}(\ddot{q}, \ddot{q}) = \dddot{\ddot{M}}(\ddot{q}) \ddot{q}, \quad \dddot{p}(\ddot{q}) = \dddot{M}(\ddot{q}) \ddot{V}(\ddot{q}) \quad (12)
\]
and
\[
\bar{w}(\bar{q}, \dot{\bar{q}}) = \bar{M}(\bar{q})\ddot{\bar{q}}(\bar{q}) + \bar{C}(\bar{q}, \dot{\bar{q}})\dot{\bar{q}}(\bar{q})
\] (13)

We set
\[
\bar{G}(\bar{q})\ddot{\bar{u}} = \bar{\tau}(\bar{q}, \dot{\bar{q}})
\]
\[
\ddot{\bar{u}} = (\bar{G}(\bar{q})^T\bar{G}(\bar{q}))^{-1}\bar{G}(\bar{q})^T\bar{\tau}(\bar{q}, \dot{\bar{q}})
\] (14)

Notice that \( E \) should be selected to be large enough so that eqn. (7) has a real solution. In equation (11) \( \gamma \) is feedback control gain, not necessarily positive which determines the convergence rate and the sense in which the desired velocity field will be followed. \( \dot{\bar{V}}(\bar{q}) \) is the gradient of desired velocity field given by
\[
\dot{\bar{V}}(\bar{q}) = \sum_{k=1}^{n+1} \frac{\partial \bar{V}(\bar{q})}{\partial q_k} \dot{q}_k
\] (15)

3. Modeling of the AUV system

We use only 2 pairs of thrusters \((P_1, P_2)\) for planar tracking. Each pair of thrusters is driven with same velocity in each direction \((P_y = P_1 = P_2 \) for y-direction and \(P_x = P_3 = P_4 \) for x-direction). For simplification, we fix the AUV movement in xy-plane without rotation (fix heading). Therefore, the AUV is fully actuated for planar tracking control (2-DOF) as shown in Fig. 1.

x-direction EOM
\[
M\ddot{x} + \frac{\rho C_{Dx} A_x \dot{x}^2}{2} = P_3 + P_4
\] (16)
\[
\ddot{x} = \frac{P_3 + P_4}{M} - \frac{\rho C_{Dx} A_x \dot{x}^2}{2M}
\] (17)

For \( P_x = P_3 = P_4 \)
\[
\dot{x} = \frac{2P_x}{89.50} - 0.787\dot{x}^2
\] (18)

y-direction EOM
\[
M\ddot{y} + \frac{\rho C_{Dy} A_y \dot{y}^2}{2} = P_1 + P_2
\] (19)
\[
\ddot{y} = \frac{P_1 + P_2}{M} - \frac{\rho C_{Dy} A_y \dot{y}^2}{2M}
\] (20)

For \( P_y = P_1 = P_2 \)
\[
\dot{y} = \frac{2P_y}{89.50} - 0.425\dot{y}^2
\] (21)

![Fig. 1. ROV’s free body diagram in xy-plane](image)

Given mass of the AUV \( M = 89.50 \) kg. Drag coefficient in x and y direction are \( C_{Dx} = 0.55 \) and \( C_{Dy} = 0.48 \). Sectional area of the AUV in x and y direction are \( A_x = 0.257 \) m\(^2\) and \( A_y = 0.159 \) m\(^2\). Water density is \( \rho = 997 \) kg/m\(^3\).

To transform the coupling control torque \((\ddot{\bar{\tau}})\) to the thrusters force of the AUV \((\ddot{\bar{u}})\) From eqn. (14) the matrix relation of each coupling torque \((\bar{G}(\bar{q}))\) can be rewritten as
\[
\bar{G}(\bar{q}) = \begin{bmatrix}
2 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (22)
4. The AUV Planar Trajectory Tracking

The path planning of the AUV was defined as a circle with 3 m. radius (r), in rectangular space 5x5 m.
for example we start from point (x,y)=(-5,-5) as shown in Fig. 2(b) and from point (x,y)=(1,1) as Fig. 2(c).
Circular contour equation is
\[ f(x, y) = x^2 + y^2 - r^2 \]  (24)
desired velocity in x-direction
\[ v_x = \dot{x} = y - x(x^2 + y^2 - r^2) \]  (25)
desired velocity in y-direction
\[ v_y = \dot{y} = -x - y(x^2 + y^2 - r^2) \]  (26)
We combine both desired velocity (v)
\[ v = [\dot{x} \; \dot{y}]^T \]  (27)
Desired velocity in eqn. (27) is fed to PD speed control to maintain the AUV’s speed.

Fig. 2. (a) Velocity field for tracing a circle on a rectangular space (b) The example path from start point (-5, -5) and (c) The example path from point (1,1)

5. Simulation and Results

EOM of the AUV (18) and (21) represent in the parts of the AUV system block. The local coordinates
\[ q = [x \; y]^T \] and the local augmenting velocities \[ \dot{q} = [\dot{x} \; \dot{y} \; \dot{q}_{n+1}]^T \] are fed to Passive Velocity Field Controller block for estimate desired velocity to track trajectory. The controller \( \ddot{u} \) (eqn. (23)) is fed-back to the augmented system. Desired velocity (v) is fed to PD speed control to control angular trajectory of the AUV. The systems are shown in Fig. 3.

Gain tuning techniques
1. Select \( E \) be large enough so that eqn. (7) have real solutions. \( (E = 5000000) \)
2. Define inertia of flywheel \( M_F > 0 \). \( (M_F = 100) \)
3. Select $\gamma$ for convergence rate. ($\gamma = -\frac{1}{450}$)

4. Set error gains of trajectory tracking ($K_x = \text{sign}(x)$ and $K_y = \text{sign}(y)$)

5. Tune PD gain $K_p$ and $K_d$ for control angular trajectory of the AUV ($K_{px} = K_{py} = 100$ and $K_{dx} = K_{dy} = 100$)

6. Set simulation time. ($t = 850$)

The results shown that the AUV converge to desired path following the velocity field, that it was designed. Fig. 4(a) and (b) are simulation results on start point (-5, -5) and (1, 1).

Fig. 4. The AUV’s trajectory in the configuration space (a) start on (-5, -5) and (b) start on (1,1)

6. Conclusions

This paper showed an application of PVFC methodology for the AUV planar trajectory tracking. PVFC is the main controller for design velocity field. The PD control was inserted to control angular trajectory of the AUV referred to desired velocity ($v$). The simulating results illustrate and verify that the PVFC could be applied with the AUV planar trajectory tracking whenever from everywhere local the AUV on space.

7. References


