A Comparative Study of performance of Blackman Window Family for Designing Cosine-Modulated Filter Bank

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Abstract. In this work, an improved approach is presented for designing the nearly perfect reconstructed cosine-modulated (CM) filter banks with given stopband attenuation and channel overlapping. The method employs Blackman window family to design the prototype filter for filter banks with novelty of exploiting spline functions in the transition band of the ideal filter instead of using the conventional brick-wall filter. The cutoff frequency is optimized using linear optimization technique such that the filter coefficients values at frequency (\( \omega = \pi / 2M \)) is approximately equal to 0.707. The simulation results illustrate significant reduction in amplitude distortion, aliasing error and computation time. In case of subband coding of the ECG signals, the proposed method with Blackman window family yields good fidelity performance measuring parameters.

Keywords: Cosine modulation, Filter banks, Pseudo QMF bank, QMF bank.

1. Introduction

Many advancements in the area of multirate filter banks in conjunction with the ever increasing numerous applications have made multirate filter banks design an increasingly important field of research. The research effort was first focused on design of two-channel quadrature mirror filter (QMF) bank [1], which was extended by several authors [2-4]. Originally a two-channel QMF bank was used for aliasing cancellation in subband coding [1]. Since then, it is used in several applications such as perceptual coding of digital audio coding, image, and biomedical signal processing due to the advances in QMF banks. Later on, its M-channel extension was developed [5, 6]. In M-channel filter bank, the analysis section consists of M-parallel band filters followed by M-fold decimator in each channel, while synthesis section consists of M-fold up-sampler in each channel, followed by a band pass filter as shown in Fig.1.

Fig.1: A generalized block diagram of M-channel cosine-modulated filter bank
Among the different classes of multi-channel filter banks, cosine-modulated filter banks are the most frequently used filter banks due to simpler design, where analysis and synthesis filter banks are derived by cosine modulating a low-pass prototype filter. The design of whole filter bank thus reduces to that of a single low-pass prototype filter. Filter banks with such properties have been studied in depth and various approaches have been successfully developed [7-12]. Creusere and Mitra [7] have introduced the first systematic iterative technique for the design of pseudo QMF banks in which, the passband edge frequency was optimized to minimize the objective function using linear optimization. This method gave better performance in terms of the peak reconstruction error (PRE). Next, Lin and Vaidyanathan [8] used Kaiser Window approach to design the prototype filter while, the authors in [9-11] have modified the method given in [7] to include other windows. Recently, the authors in [12] have used Blackman window family for subband coding of ECG signal.

Some applications such as in perceptual audio coding [13], a filter bank with high stopband attenuation, small channel overlapping, and efficient resolution switching is required for high quality reconstruction of sound. In ECG signal processing [13], especially for heart beat detection, the filter banks with fast switching resolution, and adjustable stopband attenuation is required. Therefore, there is a need of flexible method that can design the filter banks satisfying such sophisticated specifications.

In this paper, therefore, an improved approach for designing of optimized prototype filter for CM filter bank with given stopband band attenuation and channel overlapping is presented.

2. Analysis of Cosine-Modulated Filter Bank

A general \( M \)-channel critically sampled filter bank is shown in Fig. 1. Based on input /output relationship of filter bank, the reconstruction output is expressed as [14]:

\[
Y(z) = T_0(z)X(z) + \sum_{l=0}^{M-1} T_l(z)X\left(ze^{-j2\pi l/M}\right),
\]

where,

\[
T_0(z) = \frac{1}{M} \sum_{k=0}^{M-1} F_k(z)H_k(z) \quad \text{and} \quad T_l(z) = \frac{1}{M} \sum_{k=0}^{M-1} F_k(z)H_k(ze^{-j2\pi l/M}) \quad \text{for} \ l = 1, 2, ..., M - 1
\]

Here, \( T_0(z) \) is the distortion transfer function and determine the distortion caused by the overall system for the unaliased component \( X(z) \) of the input signal. \( T_l(z) \) for \( l = 1, 2, ..., M - 1 \) are called the alias transfer function, which determine how well the aliased components \( X(ze^{-j2\pi l/M}) \) of the input signal are attenuated. For perfect reconstruction, the distortion transfer function \( T_0(z) \) is to be a delay function \( z^K \) with \( K \) being an integer and \( T_l(z) = 0 \) for \( l = 1, 2, ..., M - 1 \). If these conditions are satisfied, the reconstructed output signal is an exact replica of the original input signal with some delay, that is, \( y(n) = x(n-k) \). Such systems are referred as perfect reconstructed (PR) CM filter banks, and are useful in case of lossless coding. If these conditions are partially satisfied, the filter banks are called as nearly perfect reconstructed (NPR) CM filter bank with amplitude distortion and aliasing error. NPR-CM filter banks are suitable for lossy coding where the effect of these distortions is being less than those caused by the coding distortions [14].

The impulse responses of the analysis filters \( h_k(n) \) and synthesis filters \( f_k(n) \) of PR and NPR-CM filter banks are cosine-modulated version of the prototype filter \( h(n) \) with following transfer function:

\[
H(z) = \sum_{n=0}^{N} h(n)z^{-n}
\]

where, \( N \) is order of the prototype filter for the analysis and synthesis sections. The remaining analysis and synthesis filters are determined by

\[
h_k(n) = 2h(n)\cos\left[\alpha_k \left(n - \frac{N-1}{2}\right) + \theta_k\right],
\]

\[
f_k(n) = h_k(N-1-n)
\]

with \( k = 0,1, ..., M - 1 \), and \( \alpha_k = (2k+1)\pi / 2M \)
where, $\theta_k = (-1)^k \pi / 4$ \hspace{1cm} (7)

Therefore, the design process of CM filter bank reduces to design of a single prototype filter. As mentioned earlier, three types of errors encountered in filter banks are, namely, phase distortion, amplitude distortion, and aliasing distortion. In NPR-CM filter bank of the type being considered, phase distortion is eliminated through the use of linear-phase filters. Therefore, these filter banks are characterized by the error in the amplitude response given by

$$e_{am} = \max \left(1 - |T_0(e^{j\omega})|\right)$$

The worst case aliasing distortion ($e_a$) is given by

$$e_a = \max \left(T_l(e^{j\omega})\right) \text{ for } \omega \in [0,\pi], 1 \leq l \leq M - 1$$

Whereas, total aliasing distortion is given by

$$e_a = \sqrt{\sum_{l=1}^{M-1} |T_l(e^{j\omega})|^2}$$

3. Design of Prototype filter using Window Function

For this work, Blackman window family is exploited for the design of the prototype filter for CM filter bank. A low pass prototype filter $h(n)$, of order $N$ is designed using window function and is of the form

$$h(n) = h_{id}(n)w(n),$$

where

$$h_{id}(n) = \frac{\sin(\omega_c(n - N/2))}{\pi(n - N/2)}$$

is an ideal impulse response filter with cut-off frequency ($\omega_c$), while $w(n)$ is a window function of order $N$.

For improving the performance of filter designed with window function, a function called spline, and is defined in transition region to shape it such that it allows an explicit control on transition bandwidth. In addition, it also eliminates the Gibbs’ phenomenon even more. The detailed discussion on design of filter with window function along with spline function in transition region is in [14, 16] and the references therein. Therefore, Eqn. (12) is modified due to use of spline function in transition region:

$$h_{idS}(n) = \left[\frac{\sin(\omega_c(n - N/2))}{\pi(n - N/2)}\right]^\mu \left[\frac{\sin([n - N/2](\omega_s - \omega_p)/2\mu)}{[n - N/2](\omega_s - \omega_p)/2\mu}\right]$$

where, $\omega_p$ is the passband edge frequency, and $\omega_s$ is the stopband edge frequency. While $\mu$ is the order of the spline function. For calculating order of the spline function, analytical formula is used given in [14]. Hence, the filter design using window function with spline function in transition region is specified by four parameters: order of filter, cut-off frequency, window shape parameter and spline function order.

In this method, the prescribed channel overlapping is achieved through the roll-off factor ($RF$), which influences the overlapping between different subbands. If it is less than one, each channel overlaps with its adjacent channels. The value of $RF$ between {1 to 2} gives better performance and it decides the cut-off frequency ($\omega_c$), and stopband edge frequency:

$$\omega_c = \frac{\pi RF}{4M}$$

and

$$\omega_s = \frac{(1 + RF)\pi}{2M}$$
where, $M$ is the number of channel in CM filter bank. A detailed discussion on roll-off factor is given in [13, 17] and the references there in.

From Eqns. (14) and (15), the passband edge frequency is estimated as

$$\omega_p = \frac{\pi}{2M}$$  \hspace{1cm} (17)

Then, order of prototype filter is estimated using $A_s$, $\omega_p$, and $\omega_c$ with

$$N = - \frac{(A_s - 7.95)}{14.95\Delta f}$$  \hspace{1cm} (18)

In Eqn. (18), $\Delta f = (\omega_c - \omega_p) / 2$. For window functions of the type being used, the window shape parameter is estimated using the stopband attenuation. In this work, Blackman window family [12, 18, 19] is used for designing the prototype filter.

4. Proposed Methodology

In $M$-channel cosine modulated filter bank, perfect reconstruction (PR) is possible if Eqn. (18) is satisfied.

$$\left| H_0(e^{j\omega}) \right|^2 + \left| H_0(e^{j(\omega - \pi / M)}) \right|^2 = 1, \text{ for } 0 < \omega < \pi / M$$  \hspace{1cm} (19)

If it is evaluated at $\omega = \pi / 2M$, then it is reduced to

$$\left| H_0(e^{j\pi/2M}) \right| = 0.707$$  \hspace{1cm} (20)

Hence, the design problem of CM filter bank is condensed to design a prototype filter whose magnitude response at frequency $\omega = \pi / 2M$ is 0.707. In the proposed methodology, cut-off frequency is optimized so that Eqn. (19) is satisfied.

The following steps discuss the main computational process of the proposed algorithm:

Step1: Specify stopband attenuation ($A_s$), order of spline function ($\mu$), and roll-off factor ($RF$).

Step2: Initialize counter ($Count$), Tolerance ($Tol$) and Step size ($Step$).

Step3: Estimate cut-off frequency ($\omega_c$) and order of filter with respective Eqns. (15) and (16) using given specifications.

Step4: Evaluate the prototype filter coefficient using Blackman window family with $N, \omega_c$ and $\mu$ before optimization start and the magnitude response of designed filter (MRD) at $\omega = \pi / 2M$. Also compute error or deviation of magnitude response of designed (MRD) filter from the ideal magnitude response (MR) given by Eqn. (20).

$$error = MR - MRD$$  \hspace{1cm} (21)

Step5 (a): If tolerance ($Tol$) is not satisfied, then cut-off frequency is varied in two ways:

a. If $MR > MRD$, increase $\omega_c = \omega_c + step$

b. Otherwise $\omega_c = \omega_c - step$.

Step5 (B): If $Tol$ is satisfied. Then, design the other filters using Eqns. (5) and (6).

Step6: Redesign the prototype filter using new $\omega_c$ and same order. Calculate MRD and also Error.

Step7: Increment the counter by 1.

Step8: Step=step /2. Go to step 5 till the tolerance is not satisfied.

5. Results and Discussions

In this section, three design examples are included to demonstrate the efficacy of the proposed method. The performance of this algorithm is evaluated in terms of amplitude distortion, aliasing error, computational time and number of iterations. In the following design examples, first of all comparison of Blackman window family in the proposed method is carried out. Later on, subband coding of ECG signal with this method is performed.
5.1. Design examples

Example-I: A 32-channel cosine-modulated filter bank has been designed with $A_s = 100 \, \text{dB}$, $RF = 1.82$ and order of spline function $u = 13$. For given $A_s$ and channel overlapping, a prototype filter is designed with Blackman window family. The performance parameters obtained are listed in Table I, and the simulation results obtained with Blackman window are graphically illustrated in Fig. 3.

Similarly, two more design examples have been designed using the proposed method and the simulation results are listed in Table I. As it can be seen that the design obtained with the proposed method are superior in terms of amplitude distortion, aliasing distortion, and conversed in low number of iteration and less computational time. In addition to these, it is linear and easy to implement.

5.2. Comparison of Blackman window family

The proposed method has been used to design a number of 32-channel cosine-modulated filter bank with stopband attenuation of -100dB. When different Blackman window functions are compared, the proposed method with Blackman window gives better performance in term of amplitude distortion, while Blackman-Harris (4-terms, -92dB) perform better in term of aliasing error. The average peak amplitude distortion and aliasing error resulted is $1.533 \times 10^{-3}$ in Blackman window and $1.010 \times 10^{-3}$ in Blackman-Harris (4-terms, -92dB). As can be seen from both the tables, the number of iterations and computational time required by all Blackman window functions are comparable. In overall, the proposed method provides better designs with Blackman-Harris (4-terms, -74dB).

5.3. Subband coding of ECG signal

In this sub-section, the proposed method has been used for the subband coding of ECG signals. The quality retrieved signal is evaluated by considering several fidelity assessment parameters discussed in [13, 20]. An 8-channel CM filter bank designed using proposed method with $A_s = 95 \, \text{dB}$ and $RF = 1.29$ was
exploited for subband processing of ECG signals. For this, ECG records (MIT-BIH Rec. 100: M 69) have been taken from MIT-BIH Database and the performance measures obtained in each variants of Blackman window family is listed in Table 4.

It is evident that the good fidelity measures can be achieved with the proposed method via Blackman window family within acceptable range [13]. In terms of fidelity measures, Blackman window yields improved performance parameters as compared to other variants. Hence it offers superior reconstruction quality.

**TABLE I:** Relative Performance of Blackman Window Family for Different Values of Filter Taps and Roll-off Factor

<table>
<thead>
<tr>
<th>Type of Window</th>
<th>Filter Taps</th>
<th>RF</th>
<th>Amplitude Dist.  ($e_{am}$)</th>
<th>Aliasing Error  ($e_a$)</th>
<th>NOI</th>
<th>CPU (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blackman</td>
<td>448</td>
<td>1.83</td>
<td>1.649 x 10^{-3}</td>
<td>1.231 x 10^{-6}</td>
<td>29</td>
<td>0.2808</td>
</tr>
<tr>
<td>Blackman-Harris (3-term, -67dB)</td>
<td>448</td>
<td>1.83</td>
<td>2.501 x 10^{-3}</td>
<td>8.270 x 10^{-6}</td>
<td>31</td>
<td>0.1872</td>
</tr>
<tr>
<td>Blackman-Harris (4-terms, -92dB)</td>
<td>448</td>
<td>1.83</td>
<td>4.802 x 10^{-3}</td>
<td>1.186 x 10^{-7}</td>
<td>27</td>
<td>0.1716</td>
</tr>
<tr>
<td>Blackman-Harris (4-terms, -74dB)</td>
<td>448</td>
<td>1.83</td>
<td>1.794 x 10^{-3}</td>
<td>2.036 x 10^{-7}</td>
<td>29</td>
<td>0.1872</td>
</tr>
<tr>
<td>Blackman-Harris (3-term, -67dB)</td>
<td>512</td>
<td>1.61</td>
<td>1.417 x 10^{-3}</td>
<td>6.261 x 10^{-7}</td>
<td>31</td>
<td>0.1872</td>
</tr>
<tr>
<td>Blackman-Harris (4-terms, -92dB)</td>
<td>512</td>
<td>1.61</td>
<td>2.667 x 10^{-3}</td>
<td>3.724 x 10^{-7}</td>
<td>30</td>
<td>0.2028</td>
</tr>
<tr>
<td>Blackman-Harris (4-terms, -74dB)</td>
<td>512</td>
<td>1.61</td>
<td>4.727 x 10^{-3}</td>
<td>8.436 x 10^{-8}</td>
<td>30</td>
<td>0.1872</td>
</tr>
<tr>
<td>Blackman</td>
<td>576</td>
<td>1.58</td>
<td>2.259 x 10^{-3}</td>
<td>5.733 x 10^{-3}</td>
<td>31</td>
<td>0.1872</td>
</tr>
<tr>
<td>Blackman-Harris (3-term, -67dB)</td>
<td>576</td>
<td>1.58</td>
<td>2.786 x 10^{-3}</td>
<td>9.569 x 10^{-7}</td>
<td>29</td>
<td>0.1248</td>
</tr>
<tr>
<td>Blackman-Harris (4-terms, -92dB)</td>
<td>576</td>
<td>1.58</td>
<td>4.733 x 10^{-3}</td>
<td>5.523 x 10^{-8}</td>
<td>30</td>
<td>0.1092</td>
</tr>
<tr>
<td>Blackman-Harris (4-terms, -74dB)</td>
<td>576</td>
<td>1.58</td>
<td>2.328 x 10^{-3}</td>
<td>1.278 x 10^{-8}</td>
<td>31</td>
<td>0.1248</td>
</tr>
</tbody>
</table>

**TABLE II:** A comparison of fidelity assessment parameters of Blackman window family

<table>
<thead>
<tr>
<th>Type of Window</th>
<th>PRD</th>
<th>MSE</th>
<th>ME</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blackman</td>
<td>2.270 x 10^{-2}</td>
<td>3.116 x 10^{-9}</td>
<td>1.990 x 10^{-4}</td>
<td>72.87</td>
</tr>
<tr>
<td>Blackman-Harris (3-term, -67dB)</td>
<td>4.409 x 10^{-2}</td>
<td>1.174 x 10^{-8}</td>
<td>3.724 x 10^{-4}</td>
<td>67.18</td>
</tr>
<tr>
<td>Blackman-Harris (4-terms, -92dB)</td>
<td>1.140 x 10^{-1}</td>
<td>7.923 x 10^{-8}</td>
<td>1.223 x 10^{-3}</td>
<td>58.87</td>
</tr>
<tr>
<td>Blackman-Harris (4-terms, -74dB)</td>
<td>2.961 x 10^{-2}</td>
<td>5.301 x 10^{-9}</td>
<td>2.582 x 10^{-4}</td>
<td>70.87</td>
</tr>
</tbody>
</table>

6. **Conclusions**

An improved method is presented for the design of cosine-modulated filter bank with given stopband attenuation and channel overlapping. The proposed method employs Blackman window family for designing prototype filter. Simulation results have shown that Blackman window family yields smallest reconstruction error, and aliasing error. The good fidelity measures render the proposed method highly suitable for subband coding of ECG signals.

7. **References**


